Multiregional Oligopoly with Capacity Constraints

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We develop a model of Cournot competition between capacity-constrained firms that sell a single good to multiple regions. We characterize the unique equilibrium allocation of the good across regions and provide an algorithm to compute it. We show that a reduction in transportation costs by a firm may negatively impact the profit of all firms and reduce aggregate consumer surplus if such a firm is capacity constrained. Our results imply that policies promoting free trade may have unintended consequences and reduce aggregate welfare in capacity-constrained industries. We calibrate our model to the international market of fertilizers and show that the model accurately predicts prices across regions and over time.

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1 Introduction

Many markets are dominated by a small number of firms, which compete in producing and supplying a good to different geographical regions. This imperfect multiregional competition is typical of

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many industries including energy, metals, agriculture and livestock. A prominent example is the oil market, where OPEC and the top eight non-OPEC producers make up more than 90% of the entire oil production.[1]

Firms operate under limited resources and production capacity. Those constraints force a firm to make export decisions to one region contingent on its exports to other regions, which creates linkages between regional prices. For example, if it becomes too costly to transport the good to one particular region, a firm may decide to ship the good somewhere else. The reduced competition in the former region causes the price to surge, making it a more attractive export destination for competing firms. Because of capacity constraints, the competing firms may need to divert exports from other regions, creating an endogenous correlation between regional prices of the good. The goal of our paper is to characterize the equilibrium allocation of the good and the resulting prices in this networked market. We analyze the impact of changes in import-export taxes, production costs, and production capacities on the equilibrium allocation and on aggregate welfare.

The distinguishing feature of our framework, relative to the existing literature, is that firms are capacity constrained. Exporting the good to a specific region incurs a linear cost due to shipping costs, import-export taxes, and other tariffs. We shall refer to the sum of these costs as transportation costs. Because firms operate under limited capacity, a firm may find it optimal to not compete in certain markets, hence the network of realized trade routes is endogenous in our framework.[2] We show that a reduction in import-export taxes can have qualitatively different effects on aggregate consumer surplus and firms' profits depending on whether or not the impacted firm is producing at capacity. The conventional view is that the greater the competition is, the larger is the aggregate consumer surplus; hence policies should promote the reduction or elimination of tariffs. This is true for unconstrained industries, where the reduction of import-export taxes for a firm with spare production capacity leads to a decrease in prices across all regions. However, for constrained industries, if lower import-export taxes to one region are imposed on a capacity-constrained firm, prices in other regions may increase. In a constrained setting, we show that aggregate consumer surplus rises when sales are concentrated in a few regions rather than being

[1] Apart from commodity markets, other important examples of multiregional oligopoly are the medical (Boeker et al. (1997)), airline (Evans and Kessides (1994)) telecommunication (Parker and Röller (1997)) and banking (Park and Pennacchi (2009)) industries.

[2] It is possible to restrict a firm's access to a certain region by choosing the import tariffs prohibitively high.

more homogeneously distributed across all regions. This implies that facilitating and/or opening new trade routes may negatively impact aggregate consumer surplus. We show that, if the market is capacity-constrained, opening up trading routes may also reduce the profits of *all* firms, and hence decrease aggregate welfare. These seemingly counter-intuitive results are reminiscent of the well-known Braess paradox, where the performance of a traffic network may deteriorate if network resources are increased.

In the existing literature on multiregional oligopoly without capacity constraints (e.g. Kyparisis and Qiu (1990), Qiu (1991), Abolhassani et al. (2014), and Bimpikis et al. (2019)), the equilibrium allocation of the supplied good is the solution to a complementarity problem that arises from the requirement that firms make zero marginal profits in equilibrium. In our setting, marginal profits in equilibrium can be positive for firms that are producing at full capacity, hence the equilibrium allocation can no longer be found using the same technique. Nevertheless, we are able to recover the unique equilibrium by introducing a reduced-form game, in which firms directly choose marginal profits rather than allocations of the good across markets. We show that each vector of marginal profits is associated uniquely with an allocation of goods, and that there is a one-to-one correspondence between equilibria in the original game and in the reduced-form game. Strategic complementarity of the reduced-form game leads to monotone best-response functions.[3] This implies uniqueness of the equilibrium and global stability under the tatônnement scheme.[4] Note that global stability under the tatônnement scheme generally does not hold in the original game.[5] Our tatônnement convergence result informs the design of an efficient algorithm to numerically compute the equilibrium.

Our model shows that government intervention—in the form of subsidies to its local producers—also affects foreign markets. Government subsidies lower the production cost of their local producers, making them more competitive not only in their own market but also in other markets. This

[3]Strategic complementarity is not to be confused with the complementarity problem that arises in the literature without capacity constraints. A complementarity problem seeks two non-negative vectors that satisfy a number of constraints, including the requirement that the inner product of the two vectors must equal to zero.

[4]A set of actions are an equilibrium if they are a fixed point of the best response mapping. A simultaneous discrete *tatônnement* is defined by a sequence of actions in which the current action of each firm is the best response to the previous actions of its rivals. We say an equilibrium is *globally stable* if the tatônnement converges to an equilibrium starting from any initial set of actions.

[5]In fact, in the simple setting of a single region, linear demand and constant marginal costs, the tatônnement scheme in which firms choose quantities is always unstable when the number of firms exceed three (See Theocharis (1960)).

leads to a price decrease in the regions in which these firms are competing, which may propagate to other regions through capacity-constrained producers. Firms that are producing at capacity will, as a reaction to a price decrease, reallocate their product to other regions, in which the subsidized producers are not directly selling. Subsidies to producers in one market may thus reduce the profitability of foreign producers even in their own markets. This result contributes to explain the opposition voiced by many developing countries in the World Trade Organization's Doha Round in 2001 to the agricultural subsidies provided by the United States and the European Union to their local farmers (see Subedi (2003) for additional details). Our model also shows that prices across regions are decreasing in production capacities, consistent with empirical evidence: in agricultural commodities, for example, favorable weather conditions lead to higher production capacities and hence lower regional prices for the good. On the contrary, droughts lower production capacities and thus raise prices.

We validate our framework by calibrating it to historical data in the global market for fertilizers. Our data set spans the period 2012Q1–2016Q1[6] and contains both firm-specific information (production capacities, production costs, tariffs, and shipping costs) and market-specific information (demand, willingness to pay, and prices in each region). Using historical data spanning the period 2012Q1–2014Q4, we calibrate our model and estimate the demand functions in each region. We then test the out-of-sample performance on the period 2015Q1–2016Q1. Our model is able to accurately predict prices with the absolute value of the relative error averaging 3.49%. The model's predictions are consistent with the firms' historical sales and the model is able to adequately capture firms' allocations across regions.

The rest of the paper is organized as follows. We relate our work to existing literature in Section 2. We introduce the model in Section 3. We develop the reduced-form representation of equilibria leading to the characterization of the unique equilibrium outcome in Section 4. In Section 5, we study the impact of changes in transportation costs, production costs, and production capacities on prices, aggregate consumer surplus, and aggregate welfare. We calibrate our model to the fertilizer market in Section 6. We conclude in Section 7. Appendix A contains a table summarizing our notation. Appendices B and C contain the proofs of our results. Appendix D describes the data used in the calibration, Appendix E reports the in-sample performance of our

[6]We use Q1, Q2, Q3, and Q4 to denote, respectively, the first, second, third, and fourth quarter of a year.

calibration procedure, and Appendix F shows the robustness of our calibration results to the storage parameter $S_{r,t}$.

2 Related Literature

In his seminal paper, Brander (1981) demonstrates that cross-hauling between two countries can emerge even if both countries produce a homogeneous good. In a similar setting, Bulow et al. (1985) show that in a two-region duopoly, a firm's action in one region may influence its rival's action in the other region even if regional demands are uncorrelated. These studies suggest that multiregional competition cannot be captured by models in which the demand is aggregated into a single region, and have thus sparked the construction of more general frameworks that study multiregional competition.

Harker (1986) considers a framework where multiple firms compete over spatially separated regions with at most one firm operating in each region. A similar model, and more closely related to ours, has been proposed by Nagurney (1988). She models the market by a complete bipartite graph with suppliers on one side and regions on the other. As in our paper, the weight of an edge from a firm to a region represents the per-unit transportation cost incurred by the firm for exporting the good to that region. These papers show existence and uniqueness of an equilibrium using a variational inequality approach and provide a numerical algorithm for computing it. Related studies by Kyparisis and Qiu (1990) and Qiu (1991) also consider multi-firm multiregional competition on a complete bipartite graph, focusing on theoretical properties such as continuity and differentiability of the equilibrium. In contrast to these studies, our characterization of the equilibrium allows us to determine how changes in production costs, production capacities, and import-export taxes affect regional prices and aggregate welfare.

While our paper considers competition in commodity markets, it is related to studies of competition in other industries, each modeling idiosyncrasies of their respective markets. In oligopolistic models of service industries (e.g., Lee and Cohen (1985), Li and Lee (1994), and Allon and Federgruen (2009)), customer waiting times and service level guarantees are at the heart of the analysis. Models of Cournot competition in electricity markets (e.g., Yao et al. (2008) and Ehrenmann and Neuhoff (2009)) account for the physics of electricity networks and incorporate transmission constraints in the network analysis. Like our paper, these studies consider competition across several market segments or regions in their specific setting. Our framework, in which firms are capacity constrained but transport routes are not, is designed to capture competition in networked commodity markets. The main focus of our analysis is how the interplay of transportation costs and capacity constraints affects equilibrium prices and welfare.

The paper most closely related to ours is Bimpikis et al. (2019). They consider a model, in which firms have access to some, but not all regions, and firms do not incur any transportation cost for supplying their good to different regions. They characterize the equilibrium production quantities and analyze the welfare impact stemming from firms' mergers and entries in new regions. A crucial difference between our study and theirs is that firms in our model are capacity constrained. This causes additional correlation between equilibrium prices across regions, which may revert the impact that subsidies and import-export tariffs have on welfare relative to unconstrained industries. From a methodological perspective, the presence of capacity constraints requires developing a novel equilibrium equivalence to a reduced-form model of competition, in which firms choose marginal profits as opposed to allocations across markets.

Our reduced-form model is related to Gaudet and Salant (1991). The one-to-one correspondence between marginal profits and allocations of the good is an extension of their Equation (1) to a setting of multiregional competition with capacity constraints. Different from Gaudet and Salant (1991), in the presence of capacity constraints, marginal profits are not required to be zero and are instead a strategic choice made by the firms. We solve for the equilibrium of the reduced-form model and provide an explicit construction of the corresponding equilibrium allocation. This construction is similar to that used in the single-market competition of Van Long and Soubeyran (2000). Unlike their paper, we do not assume that equilibria are non-degenerate, i.e., we allow firms to not sell at all in a specific region. This may be an innocuous assumption in the single-region setting of Van Long and Soubeyran (2000) as one can simply remove that firm from the game, but this is not the case in multiregional competition: a firm that is not competing in region A due to a geographical or political disadvantage may very well compete in region B.

Our work also relates to the stream of literature that studies *pass-through*, the transmission of changes in costs to prices. Bulow and Pfleiderer (1983) explicitly characterize the cost pass-through of a monopolist in terms of the slope of demand, slope of marginal revenue curve, and magnitude of cost change. They show that pass-through is extremely sensitive to the demand function. Seade (1985) and Dixit (1986) extend the results to an oligopoly, and show that an increase in costs can benefit all firms. The intuition is that the cost increase leads firms to produce less and charge a higher price, and the revenue obtained from the increase in price dominates the losses from decreased sales. Weyl and Fabinger (2013) develop a unifying expression that nests cost pass-though expressions in various situations including perfect competition, monopoly, Cournot competition, and differentiated products Bertrand competition. All aforementioned studies assume firms have infinite production capacity and compete in a single market. We demonstrate that there is no cost pass-through for firms producing at capacity. Moreover, we extend the analysis to a multi-regional setting and analyze the cost pass-through resulting from changes in transportation costs.

3 Model

We consider a market consisting of N firms $i = 1, \dots, N$ that produce a single homogeneous good. Firms compete in different geographical regions $r = 1, \dots, R$. Each firm i chooses an allocation $q^i = (q_1^i, \dots, q_R^i)$ of quantities, where q_r^i denotes the quantity sold in region r. We denote by κ^i the production capacity of firm i and by t_r^i firm i's transportation cost per unit of good transferred to region r. Throughout the paper, we use $Q = \left(q_r^i\right)_{\substack{i=1,\dots,N \\ r=1,\dots,R}}$ to denote the $R \times N$-dimensional matrix containing the firms' allocations as column vectors. We denote by $q_r = (q_r^1, \dots, q_r^N)$ the vector of the quantities sold in region r, we denote by $q_r^{tot} := \sum_{i=1}^n q_r^i$ the total amount of the good sold to region r, and by $q_{tot}^i := \sum_{r=1}^R q_r^i$ the total amount of the good produced by firm i.

Demand in each region r is modeled by a demand function $d_r(p_r)$, mapping the price p_r of the good in region r to demanded units. Buyers in region r are not willing to purchase the good if the price is at or above their reservation value w_r, i.e., $d_r(w_r) = 0$. The price p_r in each region r is determined by the market clearing condition: the total quantity sold must equal to the total quantity purchased, that is, $d_r(p_r) = q_r^{tot}$. We make the following assumption on the demand function and production cost.

Assumptions.

1. The demand function d_r in each region r is strictly decreasing, twice differentiable, and concave on the interval $[0, w_r]$.

2. For each firm i, the production cost is linear, i.e., it takes the form $a^i q^i_{tot}$.

Assumption 1 is common in the literature (see also Harker (1986), Qiu (1991), Anderson and Renault (2003), and Bimpikis et al. (2019)).[7] Because the demand function in each region is strictly decreasing, the market price in region r is given by the inverse demand function $p_r\left(q_r^{tot}\right) := d_r^{-1}(q_r^{tot})$. Firm i's profit is given by the revenue from sales net of transportation and production costs, i.e.,

$$\pi^i(Q) = \sum_{r=1}^{R} \left(p_r\left(q_r^{tot}\right) - t_r^i\right) q_r^i - a^i q^i_{tot}.$$

Definition 3.1.

1. A *strategy* q^i of firm i is the choice of an allocation $q^i = (q^i_1, \ldots, q^i_R)$ such that $q^i_r \geq 0$ for every region $r = 1, \ldots, R$ and $q^i_{tot} \leq \kappa^i$. The matrix $Q = \left(q_r^i\right)_{\substack{i=1,\ldots,N \\ r=1,\ldots,R}}$ is called a *strategy profile.*

2. A strategy profile Q is a *Nash equilibrium* if no firm has a profitable unilateral deviation, i.e., if for every firm i and every strategy $\tilde{q}^i$, we have $\pi^i\left(Q\right) \geq \pi^i\left(\tilde{q}^i, Q^{-i}\right)$, where Q^{-i} denotes the strategy profile played by i's opponents in Q.

The network topology of active edges, i.e., edges directed from a firm to a region in which this firm sells a positive quantity, is determined endogenously. The equilibrium outcome depends on all input parameters, including production capacities, production and transportation costs, and region specific demand functions. A scenario, in which firm i_0 is not allowed to sell in region r_0, can be parametrized by setting $t^{i_0}_{r_0} = w_{r_0}$, making exports by firm i_0 to region r_0 prohibitively expensive. We denote by $T = \left(t_r^i\right)_{\substack{i=1,\ldots,N \\ r=1,\ldots,R}}$ the matrix of transportation costs. Note that T determines the set of allowable links. For convenience, we summarize the notation used in Appendix A.

4 Equilibrium Allocation

In this section, we show that there exists a unique equilibrium allocation and provide a globally stable algorithm for computing it. This is achieved by first establishing a one-to-one correspondence between an equilibrium allocation and the corresponding vector of firms' marginal profits. We then

[7] As shown in the seminal paper of Bulow et al. (1985), concave demand functions implies that firms' sales decisions are strategic substitutes. This means that a more aggressive strategy by a firm (corresponding to a greater quantity competition in our framework) lowers the profit of other firms.

use this result to formulate a reduced-form game, in which firms directly choose their marginal profits instead of their allocations of the good. We construct a best-response operator in the reduced-form game, which has, as its unique fixed point, the vector of equilibrium marginal profits. Because equilibria of the original game correspond to equilibria of the reduced-form game, the one-to-one correspondence allows us to recover the unique equilibrium allocation.

4.1 Reduced-Form Representation

In an allocation Q, the marginal profit from firm i's sale in region r is equal to

$$g_r^i(q_r) := \frac{\partial \pi^i}{\partial q_r^i} = p_r'(q_r^{tot})q_r^i + p_r(q_r^{tot}) - t_r^i - a^i. \tag{1}$$

Because the demand function d_r in any region r is assumed to be strictly decreasing, concave, and twice differentiable, so is its inverse $p_r = d_r^{-1}$. It follows that g_r^i is differentiable and strictly decreasing in q_r.[8] The following lemma provides, for each firm i, a condition on the marginal profit in each region that has to be satisfied in equilibrium. It is an extension of Equation (1) in Gaudet and Salant (1991) to a setting with multiple regions and accounting for capacity constraints, in which marginal profits can be positive in equilibrium.

Lemma 4.1. *Fix any equilibrium allocation Q. For any firm i with $q_{tot}^i > 0$, there exists a unique $\rho^i \geq 0$ such that*

$$g_r^i(q_r) \leq \rho^i \qquad \textit{and} \qquad q_r^i(g_r^i(q_r) - \rho^i) = 0 \qquad \textit{for any } r = 1, \ldots, R. \tag{2}$$

By setting $\rho^i = 0$ for any firm i with $q_{tot}^i = 0$, this defines a map $Q \mapsto \rho$ such that (2) *is satisfied for every firm $i = 1, \ldots, n$.*

The complementary slackness condition (2) states that, in equilibrium, any firm i is either not selling to region r or its marginal profit from doing so is equal to ρ^i. This implies that the marginal profits of firm i are identical in all regions to which it is selling a positive quantity. If the marginal

[8] Let δ_{ij} denote the Kronecker delta function. Since p_r is strictly decreasing and concave, differentiating once yields

$$\frac{\partial}{\partial q_r^j} g_r^i(q_r) = p_r''(q_r^{tot})q_r^i + (1 + \delta_{ij})p_r'(q_r^{tot}) < 0.$$

profit were lower in region r_1 than in region r_2, firm i could raise its profits by reducing $q^i_{r_1}$ and increasing $q^i_{r_2}$. Henceforth, we refer to ρ^i simply as firm i's marginal profit.

Lemma 4.1 reduces the space of candidate equilibrium allocations to those that satisfy (2). The following proposition states that, for any given vector of marginal profits, there exists exactly one allocation that satisfies the necessary condition (2) for each firm.

Proposition 4.2. *Fix a vector of marginal profits $\rho \in [0, \max_r w_r]^N$. Then there exists a unique non-negative allocation $\hat{Q}(\rho)$ satisfying (2) for every firm i. The allocation can be computed using Algorithm 4.1 given below, which is guaranteed to terminate in a finite number of iterations.*

Proposition 4.2 allows us to lower the dimensionality of the space of decision variables by considering the *reduced-form* game, in which each firm chooses its marginal profit directly rather than its allocation of the good across regions. While not every allocation of the original game has a corresponding vector of marginal profits in the reduced-form game, all equilibria are preserved because they satisfy (2). The allocation $\hat{Q}(\rho)$ is the one that would attain marginal profits ρ if firms were not capacity constrained. We account for capacity constraints in Section 4.2, where we construct the equilibrium vector of marginal profits.

Algorithm 4.1. *Fix a vector of marginal profits ρ. For any region r, let σ_r be an ordering of firms according to their competitiveness in region r, that is, $\rho^{\sigma_r(i)} + t_r^{\sigma_r(i)} + a^{\sigma_r(i)}$ is non-decreasing in i. For any $x \geq 0$ and any $k \in \mathbb{N}$, define the function $G_{r,k}(x) = p'_r(x)x + kp_r(x)$. For each region $r = 1, \ldots, R$, initialize $\mathcal{I}_r^{(0)} = \emptyset$ and $q_r^{tot,(0)} = 0$ and perform the following iteration for $k \geq 1$:*

1. *If $k = N+1$ or $\rho^{\sigma_r(k)} + t_r^{\sigma_r(k)} + a^{\sigma_r(k)} \geq p_r(q_r^{tot,(k-1)})$, return $\mathcal{I}_r = \mathcal{I}_r^{(k-1)}$ and $\hat{q}_r(\rho)$ defined by*

$$\hat{q}^i_r(\rho) = \frac{\rho^i + t^i_r + a^i - p_r\left(q_r^{tot,(k-1)}\right)}{p'_r\left(q_r^{tot,(k-1)}\right)} 1_{\{i \in \mathcal{I}_r\}}, \tag{3}$$

   where $1_{\{i \in \mathcal{I}_r\}}$ equals one if $i \in \mathcal{I}_r$.

2. *Otherwise set $\mathcal{I}_r^{(k)} = \mathcal{I}_r^{(k-1)} \cup \{\sigma_r(k)\}$ and $q_r^{tot,(k)} := G_{r,k}^{-1}\left(\sum_{i \in \mathcal{I}_r^{(k)}} \rho^i + t^i_r + a^i\right)$.*

The idea underlying Algorithm 4.1 is the following. If the set $\mathcal{I}_r$ of firms that sell a positive quantity in region r was known for every region, then the system of equations (1) would be sufficient to determine the allocation.[9] However, because the sets $\mathcal{I}_r$ are not known a priori, we compute them iteratively as follows. For a chosen vector of marginal profits ρ, the quantity $\rho^i + t_r^i + a^i$ is an inverse measure of firm i's chosen competitiveness in region r: the lower this quantity is, the more firm i is willing to push the price down in region r. It is therefore impossible that, in equilibrium, firm i sells a positive quantity in region r when a more competitive firm j does not.

The set $\mathcal{I}_r$ is determined via an iterative procedure. In step k, the algorithm adds the k-th most competitive firm to the set $\mathcal{I}_r^{(k)}$ if and only if it is profitable for firm k to sell in region r, given the market prices resulting from step $k-1$. Since $G_{r,k}(x)$ determines the marginal revenue in region r when k firms supply quantity x, aggregate quantities sold are updated using $G_{r,k}^{-1}$ after the inclusion of firm k.[10] This determines the market price for the next step; and the algorithm proceeds until no additional firm has an incentive to sell in region r.

4.2 Characterization of Equilibrium Allocation

In this section, we characterize the vector of marginal profits ρ that arises in the equilibrium of the original game. We show that ρ coincides with a fixed point of the best-response operator of the reduced-form game, in which firms choose marginal profits. The latter determines quantities indirectly as stated in Proposition 4.2. We prove that the reduced-form equilibrium is unique, thereby also showing the uniqueness of the equilibrium allocation in the original game via Proposition 4.2.

In equilibrium, every firm i either produces at capacity or its marginal profits in all regions are equal to zero. Indeed, if the marginal profit of a firm i were positive in some region and the firm had excess production capacity, firm i could increase its profit by producing and selling more in that region. Before formalizing this result, we introduce the following notation: For a vector ρ of marginal profits, we denote by $\rho^{-i} = (\rho^1, \ldots, \rho^{i-1}, \rho^{i+1}, \ldots, \rho^N)$ the subvector of ρ consisting of all entries except for ρ^i. We denote by $\hat{q}^i_{tot}(\,\cdot\,; \rho^{-i}) = \sum_{r=1}^R \hat{q}^i_r(\,\cdot\,; \rho^{-i})$ the total quantity produced by firm i as a function of the allocation defined in Proposition 4.2. Define the

[9] The approach of implying sales from marginal profits is pursued in Van Long and Soubeyran (2000), who analyze single-regional competition under the assumption that every firm sells a positive quantity. Our framework generalizes theirs by endogenizing the firms' decisions to export to a specific region. It also allows us to account for correlation in prices across different regions.

[10] Observe that $G_{r,k}$ is derived from (1) by summing up the marginal revenue for k firms.

operator $\Phi = (\Phi^1, \dots, \Phi^N)$ by setting

$$\Phi^i(\rho) = \begin{cases} 0 & \text{if } 0 \leq \hat{q}^i_{tot}(0; \rho^{-i}) < \kappa^i, \\ (\hat{q}^i_{tot})^{-1}(\kappa^i; \rho^{-i}) & \text{if } \kappa^i \leq \hat{q}^i_{tot}(0; \rho^{-i}), \end{cases}$$

where $(\hat{q}^i_{tot})^{-1}(\kappa^i; \rho^{-i})$ denotes the inverse function of $\hat{q}^i_{tot}(\,\cdot\,; \rho^{-i})$ evaluated at κ^i. Note that we show in Lemma B.4.(iii) that $\hat{q}^i_{tot}(\rho^i; \rho^{-i})$ is strictly decreasing and hence its inverse exists.

The best-response operator Φ can be understood as follows. If firm i were not capacity constrained, it would best respond to ρ^{-i} by producing a total quantity $\hat{q}^i_{tot}(0; \rho^{-i})$ so that its marginal profits are zero. Thus, if firm i has sufficient capacity, its best response in the reduced-form game is to choose zero marginal profits. If firm i does not have sufficient capacity to produce $\hat{q}^i_{tot}(0; \rho^{-i})$, then it produces at maximum capacity by choosing marginal profits equal to $(\hat{q}^i_{tot})^{-1}(\kappa^i; \rho^{-i})$.

Since $\Phi^i(\rho)$ is firm i's best response to ρ^{-i}, an equilibrium in the reduced-form model is a fixed point of Φ. We show in Lemma B.4.(ii) that Φ^i is constant in ρ^i and increasing in ρ^j for every $j \neq i$, hence Φ is positively monotone. This means that the reduced-form game exhibits strategic complementarities: if one firm increases its marginal profits, its competitors will optimally respond by increasing their marginal profits. We exploit this property in the following lemma, which establishes existence and uniqueness of the reduced-form equilibrium.

Lemma 4.3. *The operator Φ has a unique fixed point ρ^*.*

Tarski's fixed point theorem establishes the existence of a least and a greatest fixed point $\underline{\rho}$ and $\bar{\rho}$ of Φ, respectively, such that $\underline{\rho}^i \leq \rho^i \leq \bar{\rho}^i$ for every firm i and every fixed point ρ of Φ. Uniqueness follows from the monotonicity properties of the allocation $\hat{Q}(\rho)$. The choice of marginal profits corresponds to the firms' willingness to depress market prices: if firms are willing to accept lower marginal profits, the competition is fiercer and market prices are lower. This implies that $\hat{q}^{tot}_r(\bar{\rho}) \leq \hat{q}^{tot}_r(\underline{\rho})$ in all regions r. However, if prices are higher in $\bar{\rho}$ than in $\underline{\rho}$, firms that have the capacity to increase their production (firms with zero marginal profits) have an incentive to do so and hence $\hat{q}^i_{tot}(\underline{\rho}) \leq \hat{q}^i_{tot}(\bar{\rho})$ for each such firm i. Every other firm is producing at capacity and, by definition, does not change the total quantity it supplies. A change from $\underline{\rho}$ to $\bar{\rho}$ thus increases net supply but decreases net demand. It follows that the market clearing condition can be satisfied in both profiles $\underline{\rho}$ and $\bar{\rho}$ only if the two are, in fact, identical.[11]

A priori, not every reduced-form equilibrium has to correspond to an equilibrium in the space of sales allocation because an equilibrium of the original game needs to be robust to unilateral deviations that violate (2). The converse is true, however, as established by the following lemma.

Lemma 4.4. *Let* $Q^* = \left(q_r^{i,*}\right)_{\substack{i=1,\ldots,N \\ r=1,\ldots,R}}$ *be an equilibrium profile and let* ρ *be its vector of marginal profits uniquely associated by Lemma 4.1, i.e.,* $Q^* = \hat{Q}(\rho)$ *as defined in Proposition 4.2. Then* ρ *is a fixed point of* Φ*.*

Together, Lemmas 4.3 and 4.4 show that any equilibrium of the original game has marginal profits equal to ρ^*. This shows uniqueness of the equilibrium via Proposition 4.2. We show existence of the equilibrium in Lemma B.1, culminating in the following theorem.

Theorem 4.5. *Let* ρ^* *denote the unique fixed point of* Φ*. The unique equilibrium is given by* $\hat{Q}(\rho^*)$*.*

We conclude this section by showing that when firms iteratively best respond to marginal profits chosen by other firms, then marginal profits converge to the unique reduced-form equilibrium ρ^*.

Proposition 4.6. *Let* $\Phi^{(n)}$ *denote the n-fold application of the best-response operator* Φ*. The sequence* $\left(\Phi^{(n)}(\rho)\right)_{n\geq 0}$ *converges to* ρ^* *as* $n \to \infty$ *for any initial vector* ρ*.*

Proposition 4.6 shows that the equilibrium ρ^* of the reduced-form game is globally stable under the tatônnement scheme (see Footnote 4 for a definition). Because the equilibrium generally does not admit an explicit expression, we can use the iterative procedure implied by Proposition 4.6 to compute ρ^*. The equilibrium allocation Q^* can then be computed by applying Algorithm 4.1 to ρ^*.

Remark 4.1. We highlight that our methodology allows the computation of equilibria for capacity-constrained industries, in which marginal equilibrium profits are positive for firms that produce at capacity. While most studies of Cournot competition have characterized equilibria with zero marginal profits, there are a few others (e.g., Harker (1986), Nagurney (1988)) that provide algorithms to numerically compute equilibria of capacity-constrained Cournot competition games using a variational inequality approach.[12] However, these algorithms are not guaranteed to converge unless the sequence of vectors of firms' actions induced by the iterative best responses form a Cauchy

[11] A formal statement and proof of the characterization of $\hat{Q}(\rho)$ with respect to ρ is provided in Lemma B.4.

[12] We refer to Shapiro (1989), Gaudet and Salant (1991), Van Long and Soubeyran (2000), and Vives (2001), and references therein, for a discussion on existence and uniqueness results for Cournot games.

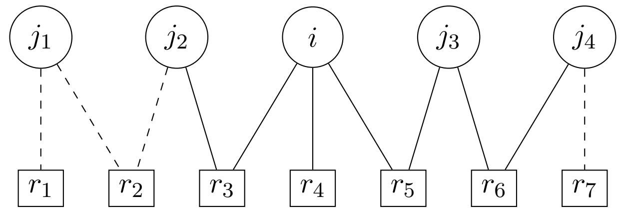

Figure 1: Suppose that firm j_3 is producing at capacity with strictly positive marginal profits and that firms j_1, j_2, and j_4 are not producing at capacity. Shown is the bipartite graph with an edge between region r and firm k if and only if $q_r^k(a) > 0$. The edge is drawn out in solid lines if the edge is contained in $\Gamma_i(a)$ and in dashed lines otherwise. The Graph $\Gamma_i(a)$ contains all firms and regions that are impacted by a change to i's production cost. This impact propagates to region r_6 and firm j_4 through the capacity-constrained firm j_3.

sequence, a condition that is difficult to verify in practice. By contrast, our algorithm converges for any set of input parameters. Moreover, our equilibrium solution yields analytical comparative static results on the allocation of the good, prices, firms' profits, and welfare.

5 Policy Implications

In this section, we investigate the impact of various policies such as subsidies, import-export taxes, and embargoes on the equilibrium allocation. While policies targeting production costs or capacities support the intuition that an increase in competition decreases prices, results pertaining to changes in transportation costs depend on the network structure and production capacities. Surprisingly, the reduction of a firm's transportation cost may negatively impact the profit of every firm and reduce aggregate consumer surplus.

5.1 Impact of Policies on Equilibrium Allocation and Prices

Financial aid given by the government to local firms in the form of subsidies has a direct role in reducing production costs. Understanding how subsidies influence the equilibrium allocation and hence also the prices across regions is critical for policy assessment.

Let $Q(a)$, $p(a)$ and $\rho(a)$ denote, respectively, the matrix of equilibrium allocations, the vector of equilibrium prices, and the vector of equilibrium marginal profits when the vector of production costs is $a = (a^1, \ldots, a^N)$. For any firm i, define the graph $\Gamma_i^0(a)$ whose nodes are firm i and the regions r, for which $q_r^i(a) > 0$. There is an edge between i and every region r supplied by firm i. For $k > 0$, let $\Gamma_i^k(a)$ denote the union of $\Gamma_i^{k-1}(a)$ with nodes j and r (as well as an edge connecting j and r) if and only if either (I) $q_r^j(a) > 0$ for $r \in \Gamma_{k-1}^i(a)$, or (II) $q_r^j(a) > 0$ with $\rho^j(a) > 0$ and

$q^j_{r'}(a) > 0$ for some $r' \in \Gamma^i_{k-1}(a)$. That is, we add all firms j that sell to regions in $\Gamma^{k-1}_i(a)$ as well as all regions, to which such a capacity-constrained firm j is selling. As the following result shows, firms and regions in the graph $\Gamma_i(a) = \lim_{k\to\infty} \Gamma^k_i(a)$ are those that are affected by a change in i's production cost. See Figure 1 for an illustration.

Proposition 5.1. *For every firm i and every vector of costs $(a^j)_{j\neq i}$, there exist two constants $\bar{a}^i, \underline{a}^i$ with $\bar{a}^i \geq \underline{a}^i \geq 0$ such that*

(i) For $a^i < \underline{a}^i$, firm i produces at capacity and $Q(a)$ and $p(a)$ are constant in a^i.

(ii) For $\underline{a}^i \leq a^i < \bar{a}^i$, the quantity $q^i_{tot}(a)$ firm i is producing is positive and strictly decreasing in a^i. On this interval, $p_r(a)$ is strictly increasing in a^i for $r \in \Gamma_i(a)$ and constant in a^i for $r \notin \Gamma_i(a)$. Moreover, $q^j_{tot}(a)$ is non-decreasing in a^i for any $j \neq i$.

(iii) For $a^i \geq \bar{a}^i$, firm i produces nothing and $Q(a)$ and $p(a)$ are constant in a^i.

When a firm i receives a subsidy, its production cost decreases. If the firm is not producing at capacity, then Proposition 5.1.(ii) shows that the resulting increase in i's competitiveness leads to a decrease of market prices in all regions in $\Gamma_i(a)$. The shock propagates to regions, in which i is not actively selling, through capacity-constrained competitors: those firms react to a decrease in prices by selling to other regions instead, increasing competition and decreasing prices in those regions as well. Because of the overall decrease in prices, no competitor of i will find it profitable to increase its sales. Thus, every firm selling to regions in $\Gamma_i(a)$ will be strictly worse off as a consequence of i's subsidies. This provides a possible explanation for the opposition of many developing countries against the agricultural subsidies provided by the US and the EU to their local farmers (see Subedi (2003) for additional details).

A large body of empirical work finds that prices do not respond to changes in production costs, and that prices react with a substantial delay (e.g., Knetter (1989), Nakamura and Zerom (2010)) In other words, pass-through of production cost into prices is incomplete and delayed. Few theoretical studies attribute this incomplete pass-through to the demand function and market structure (Atkeson and Burstein (2008)), local distributional costs (Corsetti and Dedola (2005)), and pricing policies (Bacchetta and Van Wincoop (2003)). Proposition 5.1 offers an alternative explanation based on the firms' inability to adjust production quantities due to capacity constraints.

Specifically, a firm that is producing at capacity is unable to increase production in the short-term even if costs were to decrease, hence the production cost pass-through is zero.

We also examine the effect of changes in production capacity and transportation costs on the equilibrium outcome in Appendix C.[13] We find that prices across all regions are (a) non-increasing in production capacity (Proposition C.1), and (b) non-decreasing in the transportation cost of a firm that is not producing at capacity (Proposition C.2). We note that if the transportation cost of a firm producing at capacity increases, then the equilibrium allocation and prices do not change monotonically. This has important implications on aggregate consumer surplus, which we analyze in Section 5.2.

5.2 Welfare Analysis

The aggregate consumer surplus is defined as the sum of consumer surpluses in all regions

$$CS := \sum_r \int_0^{q_r^{tot}} \big(p_r(x) - p_r(q_r^{tot})\big)dx,$$

where q_r^{tot} is the total quantity sold in region r. The following proposition characterizes the impact of changes in production cost, transportation cost, and production capacity on aggregate consumer surplus.

Proposition 5.2. *For any $i = 1, ..., N$, CS is*

(i) Non-increasing in firm i's production cost a^i.

(ii) Non-decreasing in firm i's production capacity κ^i.

(iii) Non-increasing in firm i's transportation cost t_r^i to any region r if i has excess capacity.

These claims support the intuition that, generally, greater competition leads to higher aggregate consumer surplus. However, as we show next, a reduction in transportation cost may have unintended consequences when the impacted firm is producing at capacity.

To highlight the main insights and preserve mathematical tractability, we make the simplifying assumption that regional demand functions are linear and homogeneous throughout the rest of

[13] Transportation costs quantify the cost of physical transportation, and thus also capture import tariffs and export taxes that influence the easiness of transportation. In the limiting case of vanishing or arbitrarily large transportation costs, one can capture market exits and entries.

this section.[14] Moreover, we focus on the case where, in equilibrium, each firm is active in every region. Such an assumption allows us to emphasize the prominent role played by the network of transportation costs $T = \left(t_r^i\right)_{\substack{i=1,\ldots,N \\ r=1,\ldots,R}}$. We say that an industry is *capacity constrained* if $\rho^i > 0$ for every firm i in equilibrium (recall $\rho^i > 0$ implies that firm i is producing at capacity). The remaining results in this section are stated for capacity-constrained industries.[15] We provide a comparison with an industry that is not capacity constrained in Table 1.

A key determinant in the dependence of aggregate consumer surplus and welfare on transportation costs is the accessibility of region r for firm i relative to other regions.

Definition 5.1. Given transportation costs T, the *relative accessibility* of region r for firm i is

$$A_r^i(T) := \frac{1}{R}\sum_{r'} t_{r'}^i - t_r^i.$$

We say that $A_r(T) := \sum_i A_r^i(T)$ is the *accessibility* of region r relative to the other regions and we denote by $A(T) = (A_1(T), \ldots, A_R(T))$ the vector of relative accessibilities.[16]

Note that the relative accessibility of region r for firm i is positive if and only if transportation costs to region r are lower than firm i's average transportation cost across all regions. We next introduce a measure of heterogeneity between regions' relative accessibilities, and then show that this is a critical determinant of aggregate consumer surplus in capacity-constrained industries. Specifically, for two networks of transportation costs T and $\tilde{T}$, we say that $A(T)$ *majorizes* $A(\tilde{T})$ if

$$\sum_{r=1}^{r'} A^{(r)}(T) \geq \sum_{r=1}^{r'} A^{(r)}(\tilde{T})$$

for any $r' = 1, \ldots, R$, where $A^{(r)}(T)$ is the r-th largest element of the vector $A(T)$.[17] Majorization is a measure of concentration: if $A(T)$ majorizes $A(\tilde{T})$, the regions' accessibility is more heterogeneous in network T than in $\tilde{T}$. We refer to **?** for a comprehensive treatment of majorization concepts and

[14]Linear demand functions have also been used in other studies (e.g. Singh and Vives (1984), Vives (2011)).

[15]All results (except for the second statement of Propositions 5.3 and 5.6) hold for the case where a subset of the firms are not producing at capacity.

[16]An absolute measure of region r's accessibility that is consistent with our notion of relative accessibility would be $\tau_r(T) := -\sum_i t_r^i$. In a capacity-constrained industry, however, the equilibrium allocation depends on a region r's accessibility only through $A_r(T)$: because firms have strictly positive marginal profits, increasing transportation costs across all regions by the same amount does not alter the equilibrium allocation as in part (i) of Proposition 5.1.

[17]Vector majorization requires that the sum over vector entries are identical. This is satisfied for vectors of relative accessibilities since $\sum_{r=1}^R A^{(r)}(T) = \sum_{r=1}^R A^{(r)}(\tilde{T}) = 0$ by definition.

properties. The following proposition characterizes aggregate consumer surplus effects in terms of the matrix of transportation costs.

Proposition 5.3. *In a capacity-constrained industry,*

(i) CS is increasing in t_r^i if and only if $A_r(T) < 0$. In particular, there exists $\hat{t}_r^i$ such that $\frac{\partial CS}{\partial t_r^i} > 0$ if and only if $t_r^i > \hat{t}_r^i$.

(ii) Consider two networks T and $\tilde{T}$ such that $A(T)$ majorizes $A(\tilde{T})$. Then CS is higher in T than in $\tilde{T}$.

Proposition 5.3 states that, in aggregate, consumers benefit from heterogeneity in the regions' accessibility: for a firm i that is producing at capacity, increasing the transportation costs to region r raises aggregate consumer surplus if and only if region r is already less accessible than the average region. Because firm i is producing at capacity, it reacts to the increase in transportation costs by shifting exports from region r to other regions. While this shift reduces competition and consumer surplus in region r, it increases consumer surplus in the other regions, where competition is increased. Since the remaining regions are more accessible on average than region r, exports shift from region r to regions in which consumers are gaining more on average; see Figure 2 for an illustration. We remark that capacity constraints are the key driver of Proposition 5.3. Without these constraints, firm i would not need to divert exports from region r. Thus, an increase in transportation costs to region r would reduce aggregate consumer surplus by Proposition 5.2.

Next, we study the impact of changes in transportation costs on the economy's aggregate welfare $W := CS + \Pi$, where $\Pi := \sum_j \pi^j$ is the total producer surplus. To that end, we first analyze the effect of changes in transportation costs on firms' profits. A reduction in firm i's transportation cost to region r has a direct effect on firm i's profit as the firm has to adjust its allocation across regions. In addition, there are higher-order effects due to the response by the firms' competitors to these changes in sales and the resulting impact through the network. The total impact of these effects is characterized in the following proposition.

Proposition 5.4. *In a capacity-constrained industry,*

(i) There exists $\theta > 0$ such that for any firm i, π^i is increasing in t_r^i if and only if $A_r^i < \frac{1}{N}\sum_{j \neq i} A_r^j - \theta\kappa^i$.

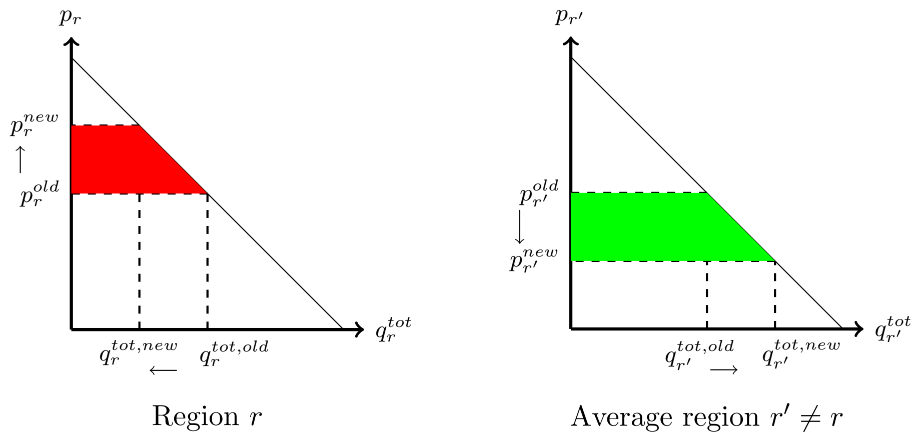

Figure 2: The left panel shows the loss in consumer surplus in region r when firm i diverts part of its exports. The right panel shows the sum of the gains in consumer surplus in all the remaining regions due to this shift in exports. Since region r is less accessible than the average region, a larger quantity is sold on average in the remaining regions. Therefore, the gain in consumer surplus in the other regions exceeds the loss in consumer surplus in region r.

(ii) π^j, $j \neq i$, is increasing in t_r^i if and only if $A_r^j > \frac{1}{N}\sum_{k\neq j} A_r^k$.

The first statement of Proposition 5.4 presents the surprising result that, in equilibrium, the profit of a capacity-constrained firm may increase with its transportation costs. Specifically, firm i's profit is increasing in t_r^i if firm i has low relative accessibility to region r. Then, as firm i diverts exports away from region r, its competitors react and allocate more of their sales to region r. Because firm i's rivals are, on average, more competitive than firm i in region r, they will move larger quantities to region r than firm i diverts away. As a result, prices increase in the remaining regions, leading to a net profit of firm i. The second statement of Proposition 5.4 shows that an increase in transportation costs of a capacity-constrained firm i to a region r increases profits of firms which are more competitive in region r than the average firm. Therefore, the more competitive firms benefit from an imbalance in the region's relative accessibility across firms. The following corollary follows directly from Proposition 5.4.

Corollary 5.5. *In a capacity-constrained industry, for any firm i, any region r, and any $(t_{r'}^j)_{(j,r')\neq(i,r)}$, there exist two thresholds $0 \leq \underline{t}_r^i \leq \bar{t}_r^i$ such that for $t_r^i \leq \underline{t}_r^i$, every firm's profit is decreasing in t_r^i and for $t_r^i \geq \bar{t}_r^i$, every firm's profit is increasing in t_r^i.*

If transportation costs of a firm i to a region r are sufficiently large so that the regions' relative accessibilities across firms are heterogeneous enough, an increase in transportation costs leads to an increase in every firm's profit. The following proposition characterizes the impact of changes in

Derivative	Network Topology	Constrained Industry	Unconstrained Industry
$\frac{\partial CS}{\partial t_r^i}$	$A_r < 0$	$+$	$-$
	$A_r > 0$	$-$	$-$
$\frac{\partial \pi^i}{\partial t_r^i}$	$A_r^i < \frac{1}{N}\sum_{j\neq i} A_r^j - \theta\kappa^i$	$+$	$-$
	$A_r^i > \frac{1}{N}\sum_{j\neq i} A_r^j - \theta\kappa^i$	$-$	$-$
$\frac{\partial \pi^j}{\partial t_r^i}\ \ \forall j \neq i$	$A_r^j > \frac{1}{N}\sum_{k\neq j} A_r^k$	$+$	$+$
	$A_r^j < \frac{1}{N}\sum_{k\neq j} A_r^k$	$-$	$+$

Table 1: Aggregate consumer surplus and firms' profits sensitivity to transportation costs in capacity-constrained and unconstrained industries. The "constrained industry" corresponds to the setting where $\rho^i > 0$ (i.e., $q_{tot}^i = \kappa^i$) for every firm i in equilibrium, and the "unconstrained industry" corresponds to the setting where $q_{tot}^i < \kappa^i$ for every firm i in equilibrium. The symbol "+" or "−" indicates, respectively, that the partial derivative in the corresponding row is "positive" or "negative" for the corresponding industry and network topology.

transportation costs, production costs and production capacity on the economy's aggregate welfare.

Proposition 5.6. *In a capacity-constrained industry,*

(i) For any firm i, any region r, and any $(t_{r'}^j)_{(j,r')\neq(i,r)}$, there exists a threshold $\hat{t}_r^i$ such that W is increasing in t_r^i if and only if $t_r^i > \hat{t}_r^i$.

(ii) W is increasing in κ^i and decreasing in a^i for any firm i.

The first statement in Proposition 5.6 follows from Propositions 5.3 and 5.4. The aggregate welfare first declines and then rises as firm i's transportation costs to region r increase, i.e., there exists a 'U' shape relationship between t_r^i and aggregate welfare. The second statement shows that the economy's aggregate welfare is increasing in production capacity and decreasing in production cost *regardless* of the network structure. We summarize the results in Table 1 and compare them with the outcomes in an industry that is not capacity constrained.

6 Model Calibration

We calibrate our model to the global market of fertilizers. Such a market is oligopolistic, with a few firms actively competing over different geographical regions. We first estimate the demand functions in the different regions from historical data. Using the calibrated model, we compute the model equilibrium allocations and prices, and then proceed to measure the model's goodness-of-fit and predictive accuracy.

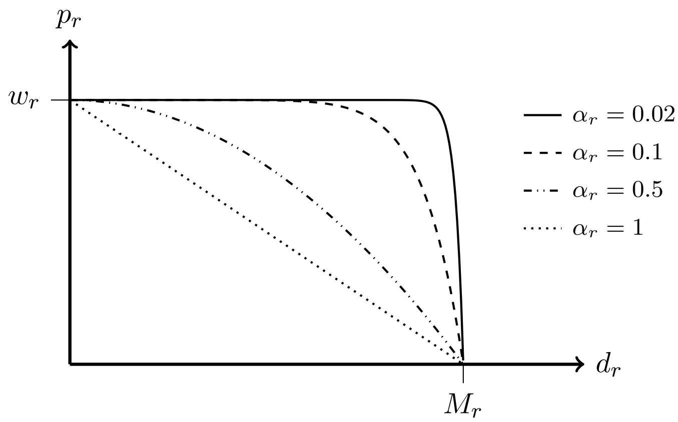

Figure 3: The inverse of the power demand function in (4) is shown for a range of elasticity parameters. The price can never exceed the buyers' willingness to pay w_r and the price at the maximum demand M_r is 0. For elasticity parameter $\alpha_r = 1$, the demand is linear between the two boundary points. As α decreases, the demand becomes more inelastic in a neighborhood of the maximum demand.

6.1 Data and Calibration Methodology

Our dataset is obtained from a major company in the fertilizer industry. There are 13 major firms in this industry, covering a wide range of geographical locations. The dataset spans the period 2012Q1 through 2016Q1 and contains information on firm-specific characteristics including production capacities, cost functions, tariffs, and shipping costs, as well as on region-specific quantities such as regional consumption and market prices. The market is divided into six regions: North America, South America, Europe, Africa, Asia, and Oceania. In our model, each period t corresponds to one quarter.

We consider regional demand functions that capture the characteristics of the different regions. Specifically, we use a family of power demand functions of the form

$$d_{r,t}(x) = M_{r,t}\left(1 - \frac{x}{w_{r,t}}\right)^{\alpha_r}, \tag{4}$$

which are parameterized by the maximum demand $M_{r,t} > 0$, the willingness to pay $w_{r,t} > 0$, and an elasticity parameter $\alpha_r \in (0, 1]$. We allow for maximum demand and willingness to pay to vary across periods. We assume the elasticity parameter to be constant across time. Figure 3 shows the effect of the elasticity parameter on the inverse demand functions.

We set $M_{r,t} = S_{r,t} + C_{r,t}$ where $C_{r,t}$ is the time series of region r's consumption in period t, and $S_{r,t}$ is the additional storage available in region r in period t. Data on aggregate historical regional

	N. America	S. America	Europe	Africa	Asia	Oceania
Elasticity	0.042	0.033	0.028	0.049	0.028	0.038

Table 2: The regional elasticity parameters that yield the lowest price root mean square error (RMSE) for the period 2012Q1–2014Q4.

consumption is obtained from the International Fertilizer Association (IFA), and is provided to member organizations. Based on the market views of our data-providing company, the additional storage $S_{r,t}$ ranges between $0.1C_{r,t}$ and $0.5C_{r,t}$. We take the average and set $S_{r,t} = 0.3C_{r,t}$. In Appendix F, we demonstrate that the estimated elasticity parameters, prices and predictions on market shares are robust to variations in $S_{r,t}$ (we consider the two corner cases, where $S_{r,t} = 0.1C_{r,t}$ and $S_{r,t} = 0.5C_{r,t}$). We estimate the willingness to pay in region r for quarter t, $w_{r,t}$, as the maximum daily observed price in region r over quarter t. We construct historical quarterly prices in each region by taking the average of daily observed prices of the good in that region over the quarter. There is no commodity market for the good we are studying, and so we rely on data from CRU Group, a privately owned business intelligence company, for data on prices. Their reported prices are based on information gathered from consulting with buyers, producers, traders, and shipping companies on confirmed deals.[18]

The data-providing company requested us to not disclose information on its identity and the precise industry in which it operates. To fulfill these requests, we use numbers of denote the different firms in the economy. To ensure replicability of the main results while protecting sensitive information, we multiply all input data by a constant $C > 0$, and report the numerical values in Appendix D.

We estimate the regional elasticity parameters via the following procedure. We use input data from the period 2012Q1–2014Q4 to find the vector of elasticity parameters α^* that produces the lowest root mean-square-error (RMSE) of the quarterly prices. The resulting vector α^* is reported in Table 2. The low regional elasticity parameters are consistent with studies of agricultural economics (e.g., see Burrell (1989) and references therein), which find that price elasticity for fertilizers is low. This can be explained by the fact that the fertilizer is an essential good with no close substitutes.

The out-of-sample predictions of our model are tested on the period 2015Q1–2016Q1 (in-sample

[18] For more details on the methodology used by CRU to determine prices, see: https://www.crugroup.com/about-cru/our-approach/methodology/).

results are given in Appendix E). We present the model's predicted quarterly prices in Table 3, and the historical quarterly prices in Table 4. Table 5 reports the relative errors of our model's predictions. The predictions are fairly accurate with the absolute value of the relative error averaging 3.49%. The model captures the price differences between regions stemming from firms' heterogeneity in transportation costs and characteristics of the regional demand functions. A direct comparison of the in-sample accuracy reported in Appendix E with the out-of-sample accuracy in Tables 3–5 indicates that our model is not overfitting the data: the in-sample RMSE is slightly lower than the out-of-sample RMSE and the average in-sample absolute value of the relative error is 2.69%.

6.2 Predictive Accuracy

We next examine the predictive power of the model in terms of the firms' export decisions. We had access to data on historical allocations only for firm 1, hence our analysis in this subsection is restricted to that firm. Figure 4 presents a comparison between firm 1's historical quarterly total volume of sales and the model's predictions for the period 2012Q1–2016Q1. Consistent with the historical data, the model predicts total volumes in the range of 300–500 kilotons per quarter. Figure 5 presents a comparison between firm 1's historical allocations and those predicted by the model, averaged over our full study period. The charts show that, overall, the allocation is captured well by our model, with relative volumes between regions aligning closely to their historical counterparts. Figure 6 presents a snapshot of the model's equilibrium allocation for all firms in 2015Q1, and provides a visual representation of the competition in the global market for fertilizers.

7 Conclusion

We study a framework of global competition between a finite number of firms. We propose a flexible model that allows for firms' heterogeneity in production costs, production capacities and transportation costs. The specific characteristics of each region are encoded in its demand function. The main distinguishing feature of our model, relative to existing studies, is that firms' production capacities are bounded. When firms are capacity constrained, changes on a firm's opportunities in one region (e.g., surge in demand, increase in willingness to pay) requires adjustments in their

Model	N. America	S. America	Europe	Africa	Asia	Oceania
2015-Q1	378.33	393.78	412.97	398.72	375.29	369.56
2015-Q2	357.52	383.40	395.02	383.93	369.82	366.02
2015-Q3	364.54	380.13	395.06	383.49	367.52	363.34
2015-Q4	353.64	357.43	380.86	366.39	350.54	349.09
2016-Q1	301.58	296.62	340.09	337.36	314.22	343.48

Table 3: The model's out-of-sample predicted prices ($/ton) for the period 2015Q1–2016Q1.

Historical	N. America	S. America	Europe	Africa	Asia	Oceania
2015-Q1	387.52	396.31	420.46	407.54	385.72	380.00
2015-Q2	366.92	390.62	406.00	398.98	384.12	377.54
2015-Q3	380.73	383.38	398.31	394.49	377.94	373.72
2015-Q4	334.19	335.29	370.29	374.29	341.83	360.00
2016-Q1	292.70	284.77	317.38	320.15	299.88	304.00

Table 4: Historical quarterly prices ($/ton) for the period 2015Q1–2016Q1.

	N. America	S. America	Europe	Africa	Asia	Oceania
2015-Q1	-2.37%	-0.64%	-1.78%	-2.16%	-2.71%	-2.75%
2015-Q2	-2.56%	-1.85%	-2.70%	-3.77%	-3.72%	-3.05%
2015-Q3	-4.25%	-0.85%	-0.82%	-2.79%	-2.76%	-2.78%
2015-Q4	5.82%	6.60%	2.85%	-2.11%	2.55%	-3.03%
2016-Q1	3.04%	4.16%	7.15%	5.38%	4.78%	12.99%

Table 5: The relative error $\frac{\text{output price}-\text{historical price}}{\text{historical price}}$ for the period 2015Q1–2016Q1.

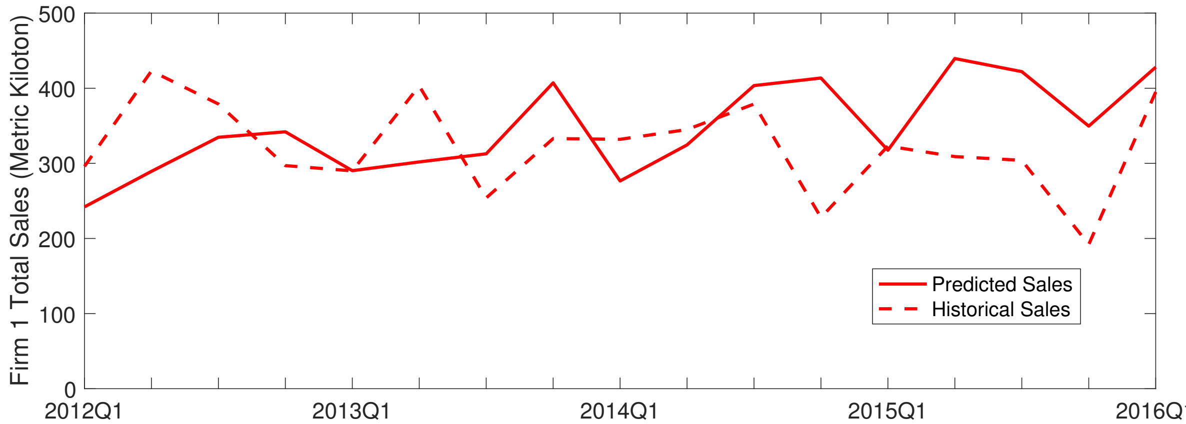

Figure 4: A comparison between firm 1's historical sales and the model's output. Sales are steady throughout the period under consideration, ranging between 300–500 tons per quarter. These sales are consistent with the model's predictions.

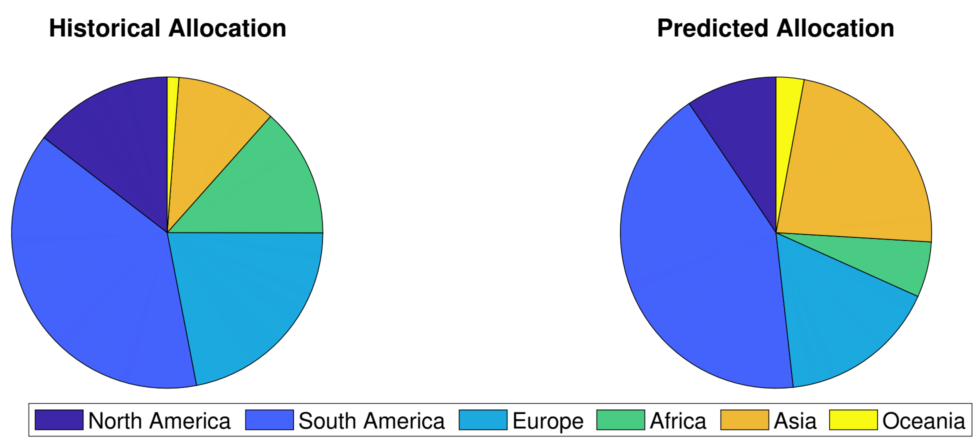

Figure 5: The charts show firm 1's historical and model's predicted allocation of exports across regions over the entire period 2012Q1–2016Q1. The overall allocation is well captured by our model: aside from the exports to Africa and Asia, the ordering of regions according to export volumes is similar under the two allocations. The data providing company informed us that the over-allocation by firm 1 in Africa, at the expense of Asia, is due to a long-term strategic decision by the firm to acquire new customers. Such long-term considerations go beyond the scope of our model.

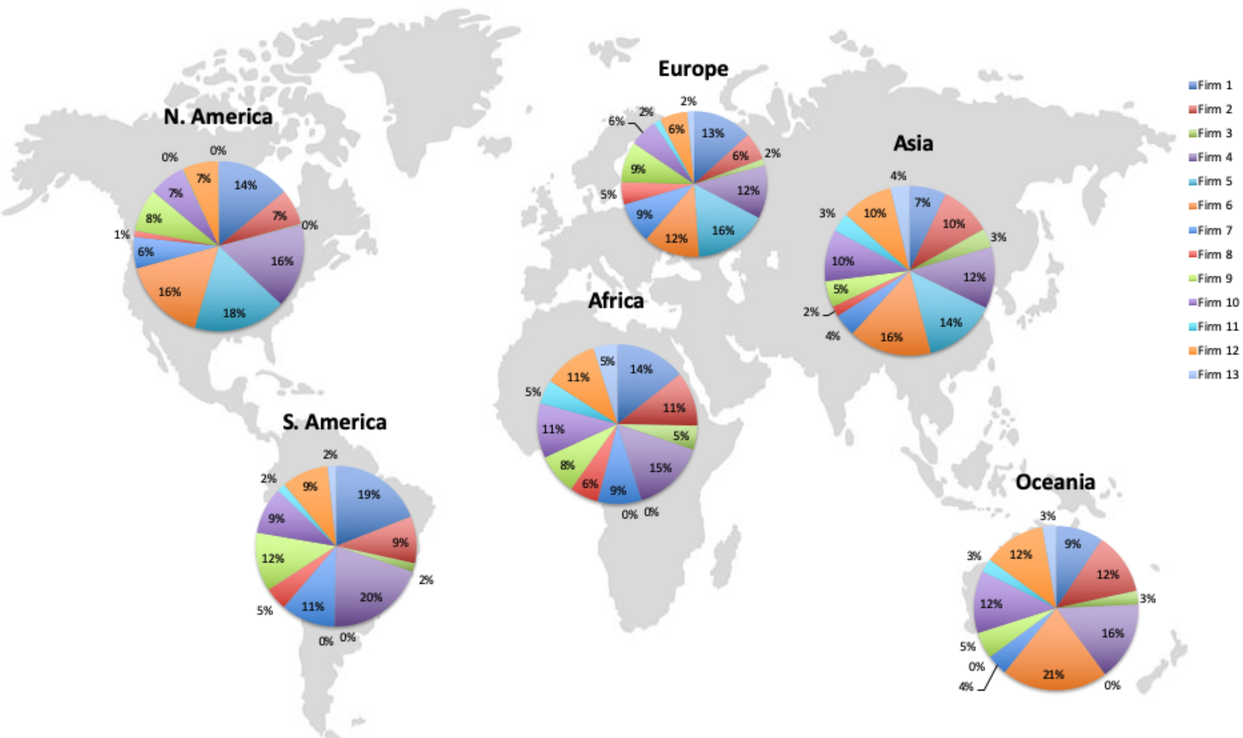

Figure 6: A snapshot of the firms' regional market share in 2015Q1.

allocation of exports to all other regions; this plays a key role in coupling prices across regions.

We develop a novel technique to analyze the market equilibrium and its properties. Our key methodological contribution is a transformation to a reduced-form representation of the model, where firms choose their marginal profits directly rather than their allocation of the good across regions. By exploiting properties of the reduced-form representation, we show the existence of a unique equilibrium allocation and we devise a globally stable algorithm for computing it. The reduced-form representation greatly lowers the dimensionality of the action space, requiring only one action for each firm regardless of the number of regions. This representation transcends the specific application considered in this paper, and can be used more broadly in other network models with constrained resource allocation. For example, design of platforms for online shopping such as those used by Airbnb and Ebay rely on matching algorithms that provide information on all candidate matches between firms and markets. In the absence of capacity constraints, this problem has been studied by Lin et al. (2017). In reality, however, firms decide which markets they should access in order to maximize their profits, given their constraints on available resources.

Our findings inform the design of welfare enhancing policies. We examine the effect of changes in production costs, production capacities and import-export taxes on regional prices and aggregate welfare. We show that changes in transportation costs have qualitatively different effects on prices depending on whether or not the impacted firm is producing at capacity. Policies that promote free trade through the reduction or elimination of tariffs (e.g., NAFTA, European Union) can actually increase prices in some regions, thereby decreasing aggregate consumer surplus. Moreover, lowering tariffs can negatively impact the profit of *every* firm. An important example for a capacity-constrained industry is the market for lithium: due to the spike in the market for electric cars, which utilize lithium-based batteries, suppliers of lithium are struggling to keep up with the demand.[19] Our calibration shows that the model is able to capture interregional dependencies between prices and sales, hence making it suitable to assess the welfare impact of international trade agreements and trade policies.

Our model can be extended along several directions. One extension is to incorporate mergers. Federal antitrust officials follow the guidelines summarized in the U.S. Department of Justice's Horizontal Merger Guideline (2010) when evaluating proposed mergers. These guidelines stress the importance of reducing the level of market concentration and increasing competition between firms. However, they do not account for the impact of mergers on equilibrium output and welfare. As we demonstrate in this paper, increasing competition by reducing transportation costs may not be welfare improving if capacity-constrained firms compete over multiple regions. Another extension is to allow firms investing in production capacity ex-ante, accounting for how such an investment would correlate regional prices ex-post.

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[19]See, for example, Lombrana and Gilbert (2017), "It's Hard to Keep Up With All That Lithium Demand," Bloomberg, August 21, 2017.

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A Notation

The following table summarizes the notation used to describe the model.

N	Number of firms
R	Number of geographic regions
q_r^i	Firm i's sales to region r
q^i	Firm i's allocation of quantities $(q_1^i, \ldots, q_R^i)$ across all regions
q_r	Vector of quantities sold to region r, i.e., $(q_r^1, \ldots, q_r^N)$
q_r^{tot}	Total amount of the good sold to region r
q_{tot}^i	Total amount of the good produced by firm i
Q	matrix of allocations by all firms, i.e., matrix with column vectors $q^1, \ldots, q^N$
$\hat{Q}(\rho)$	Capacity-unconstrained allocation satisfying (2) when marginal profits are ρ
ρ	Vector of marginal profits
κ^i	Firm i's production capacity
t_r^i	Firm i's transportation cost per unit of good transferred to region r
T	$R \times N$-dimensional matrix containing the firms' transportation costs
$d_r(\cdot)$	Demand function that maps the price of the good in region r to demanded units
$p_r(\cdot)$	Market price function in region r (i.e., inverse demand function)
w_r	Reservation price for buyers in region r
a^i	Firm i's per unit production cost
$\pi^i(\cdot)$	Firm i's profit as a function of Q
$g_r^i(\cdot)$	Function that maps the vector of sales in region r to firm i's marginal profit
$A_r^i(\cdot)$	Relative accessibility of region r for firm i as a function of transportation costs T
$A_r(\cdot)$	Relative accessibility of region r as a function of transportation costs T
$A(\cdot)$	Vector of regions' relative accessibilities as a function of transportation costs T
CS	Aggregate consumer surplus
Π	Aggregate producer surplus
W	Aggregate Welfare

B Proof of Theorem 4.5

Lemma B.1. *There exists at least one equilibrium in pure strategies.*

Proof of Lemma B.1. Observe that any firm i's action space is compact and convex and that its profit function π^i is continuous in every firm's action. The Hessian of firm i's profit function with respect to its own actions, for a fixed action profile Q^{-i} of its opponents, is given by

$$\frac{\partial^2 \pi^i(Q)}{\partial q^i_{r_1} \partial q^i_{r_2}} = \big(2p'_{r_1}(q^{tot}_{r_1}) + p''_{r_1}(q^{tot}_{r_1})\big) 1_{\{r_1 = r_2\}}.$$

The Hessian is a diagonal matrix, whose diagonal entries are negative by Assumption 1. Existence of a pure-strategy equilibrium for this concave n-person game is thus guaranteed by Debreu (1952). □

Proof of Lemma 4.1. Fix an equilibrium profile Q and a firm i. Consider first the case where $q^i_r = 0$ for every region $r = 1, \dots, R$. If there exists a region r_1, for which $0 < g^i_{r_1}(q_{r_1})$, then firm i can improve its profits by selling a positive quantity in that region. This contradicts the fact that Q is an equilibrium profile. It follows that $0 \geq \max_r g^i_r(q_r)$ and hence $\rho^i = 0$ satisfies (2). Next, consider the case where $q^i_{r_1} > 0$ for some region r_1. Set $\rho^i = g^i_{r_1}(q_{r_1})$ and observe first that $\rho^i \geq 0$ must hold as otherwise, firm i could improve its profit by reducing the quantity sold to region r_1. Suppose towards a contradiction that there exists a region r_2, for which $g^i_{r_2}(q_{r_2}) > \rho^i$. Then firm i can improve its profit by deviating to $\tilde{q}^i_{r_2} = q^i_{r_2} + \varepsilon$ and $\tilde{q}^i_{r_1} = q^i_{r_1} - \varepsilon$ for some sufficiently small $\varepsilon > 0$, contradicting the assumption that Q is an equilibrium. This shows that $g^i_r(q_r) \leq \rho^i$ for every region r and, by a symmetric argument, it also shows that $g^i_r(q_r) \geq \rho^i$ for every region r with $q^i_r > 0$. □

Proof of Proposition 4.2. We first show that for any fixed vector ρ of marginal profits, there exists an allocation Q that satisfies (2) for every firm $i = 1, \dots, N$. We establish existence by showing that such an allocation Q can be constructed by applying Algorithm 4.1 to ρ. Fix a region r and suppose that the algorithm for region r terminates after k_r steps. It is clear from the formulation of Algorithm 4.1 that $k_r \leq N + 1$. We first show that the quantities $q^{tot,(k)}_r$ are strictly increasing for $k = 1, \dots k_r - 1$. Recall, from Algorithm 4.1, that $G_{r,k}(x) = p'_r(x)x + kp_r(x)$ by definition and $G_{r,k-1}(q^{tot,(k-1)}_r) = \sum_{i \in \mathcal{I}^{(k-1)}_r} (\rho^i + t^i_r + a^i)$ by construction. It follows that

$$\begin{aligned} G_{r,k}\big(q^{tot,(k-1)}_r\big) &= p'_r(q^{tot,(k-1)}_r) q^{tot,(k-1)}_r + kp_r(q^{tot,(k-1)}_r) \\ &= G_{r,k-1}(q^{tot,(k-1)}_r) + p_r\big(q^{tot,(k-1)}_r\big) \\ &= \sum_{i \in \mathcal{I}^{(k-1)}_r} (\rho^i + t^i_r + a^i) + p_r\big(q^{tot,(k-1)}_r\big) \end{aligned}$$

for any $k < k_r$. Since $G_{r,k-1}(q_r^{tot,(k)}) = \sum_{i \in \mathcal{I}_r^{(k)}} (\rho^i + t_r^i + a^i)$ by construction, we obtain

$$G_{r,k}\big(q_r^{tot,(k)}\big) - G_{r,k}\big(q_r^{tot,(k-1)}\big) = \rho^{\sigma_r(k)} + t_r^{\sigma_r(k)} + a^{\sigma_r(k)} - p_r\big(q_r^{tot,(k-1)}\big) < 0,$$

where we have used in the last inequality that for $k < k_r$, the algorithm has not terminated yet. Because $G_{r,k}$ is strictly decreasing for any k, this shows that $q_r^{tot,(k)}$ is strictly increasing. Next, we verify that $q_r^i(\rho)$ is non-negative for every firm i. A similar argument as above shows that

$$\rho^{\sigma_r(k_r-1)} + t_r^{\sigma_r(k_r-1)} + a^{\sigma_r(k_r-1)} - p_r\big(q_r^{tot,(k_r-1)}\big) = G_{r,k_r-2}\big(q_r^{tot,(k_r-1)}\big) - G_{r,k_r-2}\big(q_r^{tot,(k_r-2)}\big) < 0,$$

where we have used in the last inequality that $q_r^{tot,(k)}$ is strictly increasing in k. Because σ_r is a non-decreasing order, this shows $\rho^{\sigma_r(k)} + t_r^{\sigma_r(k)} + a^{\sigma_r(k)} \leq \rho^{\sigma_r(k_r-1)} + t_r^{\sigma_r(k_r-1)} + a^{\sigma_r(k_r-1)} < p_r\big(q_r^{tot,(k_r-1)}\big)$ for any $k \leq k_r - 1$. Therefore, the quantity $q_r^i(\rho)$ defined in (3) is strictly positive for any firm $i \in \mathcal{I}_r$. Summing (3) over firms $i \in \mathcal{I}_r$ we obtain

$$\begin{aligned}
\sum_{i \in \mathcal{I}_r} q_r^i(\rho) &= \frac{\sum_{i \in \mathcal{I}_r} (\rho^i + t_r^i + a^i) - |\mathcal{I}_r| p_r\big(q_r^{tot,(k_r-1)}\big)}{p_r'\big(q_r^{tot,(k_r-1)}\big)} \\
&= \frac{G_{r,k_r-1}\big(q_r^{tot,(k_r-1)}\big) - (k_r-1) p_r\big(q_r^{tot,(k_r-1)}\big)}{p_r'\big(q_r^{tot,(k_r-1)}\big)} \\
&= q_r^{tot,(k_r-1)},
\end{aligned}$$

which shows that $g_r^i(q_r(\rho)) = \rho^i$ for $i \in \mathcal{I}_r$ by (3). Finally, the termination condition of the algorithm implies $p_r\big(q_r^{tot,(k_r-1)}\big) \leq \rho^i + t_r^i + a^i$ for any $i \notin \mathcal{I}_r$. Therefore, $g_r^i(q_r) = p_r\big(q_r^{tot,(k_r-1)}\big) - t_r^i - a^i \leq \rho^i$ for any such firm i. Because r was arbitrary, this shows that (2) is satisfied for every firm.

Next, we show that for any vector of marginal profits ρ the corresponding allocation Q that satisfies (2) is unique. Fix a vector ρ of marginal profits and let Q be any allocation satisfying (2) for every firm $i = 1, \ldots, N$. For any region r, let $\mathcal{I}_r(Q) := \big\{ i \;\big|\; q_r^i > 0 \big\}$ denote the set of firms that sell a positive quantity in that region. Solving $g_r^i(q_r) = \rho^i$ for q_r^i yields

$$q_r^i = \frac{\rho^i + t_r^i + a^i - p_r(q_r^{tot})}{p_r'(q_r^{tot})} \tag{5}$$

for any firm $i \in \mathcal{I}_r(Q)$. Because p_r is decreasing, it follows that $p_r(q_r^{tot}) > \rho^i + t_r^i + a^i$ for any such firm i. For a firm $i \notin \mathcal{I}_r(Q)$, condition (2) implies that $p_r(q_r^{tot}) - t_r^i - a^i = g_r^i(q_r) \leq \rho^i$ and hence

$$\mathcal{I}_r(Q) = \{i \mid \rho^i + t_r^i + a^i < p_r(q_r^{tot})\}. \tag{6}$$

Summing $g_r^i(q_r) = \rho^i$ over $i \in \mathcal{I}_r(Q)$, we obtain that

$$G_{r,|\mathcal{I}_r(Q)|}(q_r^{tot}) = \sum_{i \in \mathcal{I}_r(Q)} (\rho^i + t_r^i + a^i). \tag{7}$$

Let now Q and $\tilde{Q}$ be two allocations that satisfy (2) for every firm $i = 1, \ldots, N$. Fix a region r and suppose without loss of generality that $\tilde{q}_r^{tot} \leq q_r^{tot}$. It follows from (6) that $\mathcal{I}_r(Q) \subseteq \mathcal{I}_r(\tilde{Q})$. Suppose towards a contradiction that $\mathcal{I}_r(\tilde{Q})$ is strictly larger than $\mathcal{I}_r(Q)$. The definition of $G_{r,k}$ and the identity (7) imply that $G_{r,|\mathcal{I}_r(Q)|}\big(\tilde{q}_r^{tot}\big) = \sum_{i \in \mathcal{I}_r(\tilde{Q})} (\rho^i + t_r^i + a^i) - \big(|\mathcal{I}_r(\tilde{Q})| - |\mathcal{I}_r(Q)|\big) p_r\big(\tilde{q}_r^{tot}\big)$. Using (7) once more, we obtain

$$G_{r,|\mathcal{I}_r(Q)|}\big(\tilde{q}_r^{tot}\big) - G_{r,|\mathcal{I}_r(Q)|}(q_r^{tot}) = \sum_{i \in \mathcal{I}_r(\tilde{Q}) \setminus \mathcal{I}_r(Q)} (\rho^i + t_r^i + a^i) - \big(|\mathcal{I}_r(\tilde{Q})| - |\mathcal{I}_r(Q)|\big) p_r\big(\tilde{q}_r^{tot}\big) < 0,$$

where we have used in the last inequality that $p_r\big(\tilde{q}_r^{tot}\big) > \rho^i + t_r^i + a^i$ for any $i \in \mathcal{I}_r(\tilde{Q})$ by (6). Since $G_{r,k}$ is strictly decreasing, it follows that $\tilde{q}_r^{tot} > q_r^{tot}$, which is a contradiction. Therefore, $\mathcal{I}_r(\tilde{Q}) = \mathcal{I}_r(Q)$ must hold and hence (7) implies that $q_r^{tot} = G^{-1}_{r,|\mathcal{I}_r(Q)|}\big(\sum_{i \in \mathcal{I}_r(Q)} \rho^i + t_r^i + a^i\big) = \tilde{q}_r^{tot}$. Finally, it follows from (5) that the entire allocation $\tilde{q}_r$ must coincide with q_r. $\square$

Before proving Lemma 4.3, we give four auxiliary lemmas. For the sake of brevity, denote by $\hat{q}^i_{tot}(\rho) := \sum_{r=1}^R \hat{q}_r^i(\rho)$ the total quantity sold by firm i and by $\hat{q}_r^{tot}(\rho) := \sum_{i=1}^N \hat{q}_r^i(\rho)$ the total quantity sold in region r. The next lemma gives continuity properties of $\hat{q}_r^i(\rho)$ and $\hat{q}_r^{tot}(\rho)$.

Lemma B.2. *For any region r and any firm i, $\hat{q}_r^i(\rho)$ is continuous. By linearity, also $\hat{q}^i_{tot}(\rho)$ and $\hat{q}_r^{tot}(\rho)$ are continuous for any firm i and any region r, respectively.*

Proof. Fix a region r and set $\eta_r^i(q_r, \rho) = \max\big((g_r^i)^{-1}(\rho^i; q_r^{-i}), 0\big)$ for every $i = 1, \ldots, N$. The vector-valued function $\eta_r(\,\cdot\,, \rho) = \big(\eta_r^1(\,\cdot\,, \rho), \ldots, \eta_r^N(\,\cdot\,, \rho)\big)$ is continuous and has a unique fixed point by Proposition 4.2. Define the function $g(q_r, \rho) = \eta_r(q_r, \rho) - q_r$ and observe that $g(\,\cdot\,)$ is continuous and, for any ρ, $g(\,\cdot\,,\rho)$ has a unique zero $\hat{q}_r(\rho)$ corresponding to the unique fixed point of $\eta_r(\,\cdot\,,\rho)$. Therefore, the graph of $\hat{q}_r(\,\cdot\,)$ can be written as the set $\mathcal{A} = \{(q_r, u) \mid g(q_r, u) = 0\}$. Note that $\mathcal{A}$ is closed by continuity of g, which implies via the closed graph theorem that $\hat{q}_r(\cdot)$ is continuous. $\square$

Lemma B.3. *For any vector of marginal profits ρ and any region r, let $\hat{q}_r(\rho)$ be the unique allocation satisfying* (2)*. Then $\mathcal{I}_r(\rho) := \{i \mid \hat{q}_r^i(\rho) > 0\}$ satisfies the following single-crossing property: for every ρ^{-i}, there exists ρ_r^i such that $i \in \mathcal{I}_r(x, \rho^{-i})$ if and only if $x < \rho_r^i$.*

Proof. Fix a firm i and a vector of marginal profits ρ^{-i}. For any region r, define the smallest marginal profit $\rho_r^i = \min\{0 \le x \le w_r \mid \hat{q}_r^i(x, \rho^{-i}) = 0\}$ for which firm i does not sell a positive quantity in region r. Note that the minimum is taken over a non-empty set because $\hat{q}_r^i(w_r, \rho^{-i}) = 0$ by (6) and the minimum is attained because $\hat{q}_r^i(\cdot)$ is continuous by Lemma B.2. By construction, $i \in \mathcal{I}_r(x, \rho^{-i})$ for any $x < \rho_r^i$ as otherwise ρ_r^i would not be the minimum. Because $\hat{q}_r^i(\rho_r^i, \rho^{-i}) = 0$, it follows that $\hat{q}_r(\rho_r^i, \rho^{-i})$ satisfies (2) also for any (x, ρ^{-i}) with $x > \rho_r^i$. Proposition 4.2 thus implies that $\hat{q}_r(x, \rho^{-i}) = \hat{q}_r(\rho_r^i, \rho^{-i})$, hence $i \notin \mathcal{I}_r(x, \rho^{-i})$ for any $x > \rho_r^i$. $\square$

Lemma B.4.

(i) For any region r, $\hat{q}_r^{tot}(\rho)$ is non-increasing in ρ^i and it is strictly decreasing in ρ^i for $i \in \mathcal{I}_r(\rho)$.

(ii) For any firm $j \neq i$ and any region r, $\hat{q}_r^j(\rho)$ is non-decreasing in ρ^i. It follows from linearity that also $\hat{q}_{tot}^j(\rho)$ is non-decreasing in ρ^i.

(iii) For any firm i and any region r, $\hat{q}_r^i(\rho)$ is non-increasing in ρ^i. Moreover, $\hat{q}_{tot}^i(\rho)$ is non-increasing in ρ^i and it is strictly decreasing in ρ^i where $\hat{q}_{tot}^i(\rho) > 0$.

Proof. We have shown in the proof of Proposition 4.2 that $\hat{q}_r^{tot}(\rho)$ can be expressed as

$$\hat{q}_r^{tot}(\rho) = G_{r,|\mathcal{I}_r(\rho)|}^{-1}\left(\sum_{k \in \mathcal{I}_r(\rho)} \rho^k + t_r^k + a^k\right), \tag{8}$$

where $G_{r,|\mathcal{I}_r(\rho)|}(x) = p_r'(x)x + |\mathcal{I}_r| p_r(x)$ is differentiable and strictly decreasing in x. This implies that $\hat{q}_r^{tot}(\rho)$ is differentiable and non-increasing in ρ^i where $\mathcal{I}_r(\rho)$ is constant. Since $\hat{q}_r^{tot}(\rho)$ is continuous by Lemma B.2 and $\mathcal{I}_r(\rho)$ is constant except on a set with measure 0 by Lemma B.3, we conclude that $\hat{q}_r^{tot}(\rho)$ is non-increasing everywhere.[20] Finally, since $G_{r,|\mathcal{I}_r(\rho)|}$ is strictly decreasing and the set $\{\rho^i \mid i \in \mathcal{I}_r(\rho)\}$ is open by Lemma B.3, $\hat{q}_r^{tot}(\rho)$ is strictly decreasing in ρ^i for $i \in \mathcal{I}_r(\rho)$.

To prove the second statement, fix a firm i and let $R_i := \{\rho \mid i \in \mathcal{I}_r(\rho)\}$. Consider first the case where ρ is in the closure of R_i, that is, either i is producing a positive quantity at ρ or in arbitrarily small neighborhoods of ρ. Observe that (5) holds if and only if $\rho \in R_i$. Taking the weak derivative of (5) with respect to ρ^j for $j \neq i$ yields

$$\frac{\partial \hat{q}_r^i(\rho)}{\partial \rho^j} = -\frac{\partial \hat{q}_r^{tot}(\rho)}{\partial \rho^j}\left[1 + \frac{\left(\rho^i + t_r^i + a^i - p_r(\hat{q}_r^{tot}(\rho))\right)p_r''\left(\hat{q}_r^{tot}(\rho)\right)}{\left(p_r'(\hat{q}_r^{tot}(\rho))\right)^2}\right] \geq 0. \tag{9}$$

The inequality follows from the concavity of the demand function ($p_r'' \leq 0$), the fact that $\rho \in R_i$ implies $\rho^i \leq p_r - t_r^i - a^i$ via (5), and because the weak derivative $\frac{\partial \hat{q}_r^{tot}(\rho)}{\partial \rho^j}$ is non-positive almost everywhere by the first statement. This shows that $\hat{q}_r^i(\rho)$ is increasing in ρ^j.

For the final statement, by symmetry of part (ii) of this lemma, any $\hat{q}_r^j(\rho)$ for $j \neq i$ is increasing in ρ^i, hence

$$\hat{q}_r^i(\rho) = \hat{q}_r^{tot}(\rho) - \sum_{j \neq i} \hat{q}_r^j(\rho) \tag{10}$$

is decreasing in ρ^i. Finally, if ρ is not in the closure of R_i, then i produces 0 in a neighborhood of ρ and hence $\frac{\partial \hat{q}_r^i}{\partial \rho^i} = \frac{\partial \hat{q}_r^i}{\partial \rho^j} = 0$. Note that $\hat{q}_{tot}^i(\rho) > 0$ implies that $\hat{q}_{r_0}^i(\rho) > 0$ for some region r_0. Therefore, $\hat{q}_{r_0}^{tot}(\rho)$ is strictly decreasing in ρ^i by part (i) of this lemma. Identity (10) together with (9) show that $\hat{q}_{r_0}^i(\rho)$ is strictly decreasing in ρ^i. Since $\hat{q}_r^i(\rho)$ is non-increasing in ρ^i for all $r \neq r_0$ by the first part of this statement, it follows that $\hat{q}_{tot}^i(\rho) = \sum_r \hat{q}_r^i(\rho)$ is strictly decreasing. $\square$

Let $\bar{w} := \max_r w_r$ and observe that Φ maps $[0, \bar{w}]^N$ into itself.

Lemma B.5. *The operator Φ has a smallest and a largest fixed point $\underline{\rho}$ and $\bar{\rho}$, respectively. That is, for any fixed point ρ of Φ, $\underline{\rho}^i \leq \rho^i \leq \bar{\rho}^i$ for every $i = 1, \ldots, N$. Moreover, an iterated application of Φ to $(0, \ldots, 0) \in \mathbb{R}^N$ and $(\bar{w}, \ldots, \bar{w}) \in \mathbb{R}^N$ converges to $\underline{\rho}$ and $\bar{\rho}$, respectively.*

Proof. Note that Φ is positively monotone: Φ^i is constant in ρ^i and increasing in ρ^j for every $j \neq i$

[20] Continuity together with differentiability almost everywhere imply that $\hat{q}_r^{tot}(\rho)$ is weakly differentiable, that is, it admits functions (called weak derivatives) that integrate to $\hat{q}_r^{tot}(\rho)$. Weak derivatives can take any value at points where $\hat{q}_r^{tot}(\rho)$ fails to be differentiable, but these values do not matter because they occur only at points with measure 0. In particular, let $f_i(\rho)$ be a weak derivative of $\hat{q}_r^{tot}(\rho)$ with respect to ρ^i and choose $\rho_0^i < \rho_1^i$. Then $\hat{q}_r^{tot}(\rho_1^i, \rho^{-i}) - \hat{q}_r^{tot}(\rho_0^i, \rho^{-i}) = \int_{\rho_0^i}^{\rho_1^i} f(\rho)\,\mathrm{d}\rho^i \leq 0$ because $f_i(\rho) \leq 0$ almost everywhere. This shows that $\hat{q}_r^{tot}(\rho)$ is non-increasing in ρ^i.

as a consequence of Lemma B.4. Because Φ maps $[0,\bar{w}]^N$ into itself, Tarski's fixed-point theorem applies, which establishes the statement. $\square$

We are now ready to prove Lemma 4.3.

Proof of Lemma 4.3. Note that Lemma B.5 gives an easily verifiable condition on whether a fixed point of Φ is unique. Specifically, a fixed point of Φ is unique if and only if the two profiles $\underline{\rho}$ and $\bar{\rho}$ coincide. Let $\underline{\rho}$ and $\bar{\rho}$ denote the smallest and largest fixed points of Φ, respectively, that exist by Lemma B.5. Let $\mathcal{I} := \{i \mid \bar{\rho}^i > \underline{\rho}^i\}$ and suppose towards a contradiction that $\mathcal{I} \neq \emptyset$. It follows straight from the definition of Φ that for any fixed point ρ of Φ, either $\rho^i = 0$ or $\hat{q}^i_{tot}(\rho) = \kappa^i$. Since $\bar{\rho}^i > \underline{\rho}^i \geq 0$ for any $i \in \mathcal{I}$, it is necessary that $\hat{q}^i_{tot}(\bar{\rho}) = \kappa^i$ for those firms. Part (ii) of Lemma B.4 implies that $\hat{q}^i_{tot}(\bar{\rho}) \geq \hat{q}^i_{tot}(\underline{\rho})$ for any $i \notin \mathcal{I}$ so that the total quantity produced in $\bar{\rho}$ is at least as large as the total quantity produced in $\underline{\rho}$, which we denote by $\hat{q}^{tot}(\bar{\rho}) \geq \hat{q}^{tot}(\underline{\rho})$.

Let $\mathcal{R} = \{r \mid \sum_{i\in\mathcal{I}} \hat{q}^i_r(\rho) > 0\}$ denote the set of regions, where at least one firm from the set $\mathcal{I}$ sells a positive quantity. It follows from Part (i) of Lemma B.4 that $\hat{q}^{tot}_r(\bar{\rho}) < \hat{q}^{tot}_r(\underline{\rho})$ for every region $r \in \mathcal{R}$ and $\hat{q}^{tot}_r(\bar{\rho}) \leq \hat{q}^{tot}_r(\underline{\rho})$ for every region $r \notin \mathcal{R}$. Therefore, the total quantity sold in $\bar{\rho}$ is strictly smaller than the total quantity sold in $\underline{\rho}$, which violates the market-clearing condition. $\square$

Proof of Lemma 4.4. Fix an equilibrium allocation Q^* and let $\rho \geq 0$ denote the unique vector of marginal profits associated with Q^* specified by Lemma 4.1. By Proposition 4.2, $\hat{Q}(\rho)$ is the unique allocation with this property so that $Q^* = \hat{Q}(\rho)$. Suppose towards a contradiction that ρ is not a fixed point of Φ. Then there exists a firm i such that $\Phi^i(\rho) \neq \rho^i$. Consider first the case where $\Phi^i(\rho) > \rho^i$. It follows from the definition of Φ that either $\Phi^i(\rho) = 0$ or $\hat{q}^i_{tot}(\Phi^i(\rho);\rho^{-i}) = \kappa^i$. Since $\Phi^i(\rho) > \rho^i \geq 0$ and $\hat{q}^i_{tot}(\cdot)$ is strictly decreasing by part (iii) of Lemma B.4, we conclude that $\hat{q}^i_{tot}(\rho) > \hat{q}^i_{tot}(\Phi^i(\rho);\rho^{-i}) = \kappa^i$, a contradiction to feasibility of Q^*. Suppose next that $\Phi^i(\rho) < \rho^i$. Monotonicity of $\hat{q}^i_{tot}(\cdot)$ yields that $\hat{q}^i_{tot}(\rho) < \hat{q}^i_{tot}(\Phi^i(\rho);\rho^{-i}) \leq \kappa^i$. Because $\rho^i > \Phi^i(\rho) \geq 0$, it follows that firm i can increase its profits by increasing its production and selling a larger quantity in any region, contradiction the fact that Q^* is an equilibrium. $\square$

Proof of Theorem 4.5. Because the firms' profit functions are concave, there exists at least one equilibrium in pure strategies by Debreu (1952); see Lemma B.1 for details. Let Q^* be any such equilibrium profile. Then Lemma 4.1 asserts the existence of a vector ρ of marginal profits that satisfies (2). From Lemma 4.4 it follows that ρ is a fixed point of Φ and hence $\rho = \rho^*$ by Lemma 4.3. Finally, Proposition 4.2 shows that $\hat{Q}(\rho^*)$ is the unique allocation with marginal profits ρ^* and hence $Q^* = \hat{Q}(\rho^*)$. $\square$

Proof of Proposition 4.6. Fix an arbitrary $\rho \in [0, \bar{w}]^N$ and set $\underline{\rho} = (0, \ldots, 0)$ and $\bar{\rho} = (\bar{w}, \ldots, \bar{w})$. Let $\underline{\rho}^{(k)}$, $\rho^{(k)}$, and $\bar{\rho}^{(k)}$ denote the k-fold application of Φ to $\underline{\rho}$, ρ, and $\bar{\rho}$, respectively. Since $\underline{\rho} \leq \rho \leq \bar{\rho}$, monotonicity of Φ implies that $\underline{\rho}^{(k)} \leq \rho^{(k)} \leq \bar{\rho}^{(k)}$ for every $k \geq 1$. Since $(\underline{\rho}^{(k)})_{k \geq 1}$ and $(\bar{\rho}^{(k)})_{k \geq 1}$ converge to ρ^* by Lemmas 4.3 and B.5, it follows that $(\rho^{(k)})_{k \geq 1}$ converges to ρ^* as well. $\square$

C Proofs of Section 5

In this appendix, we provide the sensitivity analysis of prices and the equilibrium allocation with respect to capacity constraints and production costs, as well as the proofs of the results in Section 5.

We begin by stating the analogues to Proposition 5.1. Let $Q(\kappa)$, $p(\kappa)$ and $\rho(\kappa)$ denote, respectively, the matrix of equilibrium allocations, the vector of equilibrium prices, and the vector of equilibrium marginal profits when the vector of firms' production capacities is equal to $\kappa = (\kappa^1, \ldots, \kappa^N)$. For any firm i, define $\Gamma_i^k(\kappa)$, for $k \geq 0$, and $\Gamma_i^k(\kappa)$ the same way $\Gamma_i^k(a)$, for $k \geq 0$, and $\Gamma_i(a)$ are defined in Section 5.1.

Proposition C.1. *For every firm i and every vector of production capacities $(\kappa^j)_{j \neq i}$, there exist a constant $\underline{\kappa}^i \geq 0$ such that*

(i) For $\kappa^i > \underline{\kappa}^i$, firm i is not producing at capacity and $Q(\kappa)$ and $p(\kappa)$ are constant in κ^i.

(ii) For $0 \leq \kappa^i \leq \underline{\kappa}^i$, firm i is producing at capacity and $q^i_{tot}(\kappa)$ is strictly increasing in κ^i. On this interval, $p_r(a)$ is strictly decreasing in κ^i for $r \in \Gamma_i(\kappa)$ and constant in κ^i for $r \notin \Gamma_i(\kappa)$. Moreover, $q^j_{tot}(\kappa)$ is non-increasing in κ^i for any $j \neq i$.

Clearly, if firm i does not fully utilize its capacity, it will not benefit from additional capacity. Firm i that is producing at capacity, however, may increase sales if its production capacity were to increase. This is because its marginal profit can be strictly positive, and hence selling more would increase its profits. In response, rival firms will not find it profitable to increase their sales, and prices decrease in coupled regions (i.e., regions in $\Gamma_i(\kappa)$). Production capacities are especially important in agricultural commodities where crops vary from one year to the next depending on weather conditions. For example, in 2017, high corn and soybean production resulted in low market prices; see Schnitkey (2018). In contrast, droughts in Australia in 2006-2008 led to low levels of cereal stock and hence high prices; see Iizumi and Ramankutty (2015).

In December 2015 at the World Trade Organization's conference in Nairobi members agreed to abolish agricultural export subsidies by the end of 2020 (see Wilkinson et al. (2016)). The implications of such a decision can be quantified by analyzing the dependence of the equilibrium outcome on transportation costs. Let $Q(T)$, $p(T)$ and $\rho(T)$ denote, respectively, the matrix of equilibrium allocations, the vector of equilibrium prices, and the vector of equilibrium marginal profits when the matrix of firms' transportation costs is equal to T. For any firm i, define $\Gamma_i^k(T)$, for $k \geq 0$, and $\Gamma_i^k(T)$ the same way $\Gamma_i^k(a)$, for $k \geq 0$, and $\Gamma_i(a)$ are defined in Section 5.1. Let T_{-r}^{-i} denote the matrix of transportation costs without the element t_r^i.

Proposition C.2. *For every firm i and every matrix of transportation costs T_{-r}^{-i}, there exist two constants $\bar{t}_r^i, \underline{t}_r^i$ with $\bar{t}_r^i \geq \underline{t}_r^i \geq 0$ such that*

(i) For $t_r^i < \underline{t}_r^i$, firm i produces at capacity.

(ii) For $\underline{t}_r^i \leq t_r^i < \bar{t}_r^i$, the quantity $q_{tot}^i(T)$ firm i is producing is positive and strictly decreasing in t_r^i. On this interval, $p_{r'}(T)$ is strictly increasing in t_r^i for $r' \in \Gamma_i(T)$ and constant in t_r^i for $r' \notin \Gamma_i(T)$. Moreover, $q_{tot}^j(T)$ is non-decreasing in t_r^i for any $j \neq i$.

(iii) For $t_r^i \geq \bar{t}_r^i$, firm i sells nothing to region r and $Q(a)$ and $p(a)$ are constant in t_r^i.

Proposition C.2 analyzes the impact of changes in transportation cost for a firm that is not producing at capacity. The proposition shows if the transportation cost of a firm not producing at capacity decreases, such a firm will increase its aggregate sale across regions. Rival firms will not find it profitable to increase their sales, and prices decrease in regions $r' \in \Gamma_i(T)$ as the shock propagates through capacity-constrained firms.

To prove Propositions 5.1, C.1, and C.2, we will use continuity of the equilibrium allocation and the equilibrium marginal profits.

Lemma C.3. *The unique equilibrium allocation Q and equilibrium vector of marginal profits ρ are continuous in a^i, κ^i and t_r^i.*

Proof of Lemma C.3. Observe that $\Phi(\,\cdot\,, a^i)$ is continuous and has a unique fixed point by Proposition 4.6. Define $g : [0, \max_r w_r]^N \times [0, \max_r w_r] \to [0, \max_r w_r]^N$ by $g(\rho, a^i) = \Phi(\rho, a^i) - \rho$. Then, $g(\,\cdot\,)$ is continuous and, for any a^i, $g(\,\cdot\,, a^i)$ has a unique zero $\rho(a^i)$ corresponding to the unique fixed point of $\Phi(\,\cdot\,, a^i)$. Consider set $\mathcal{A} = \{(\rho, u) : g(\rho, u) = 0\} \subset [0, \max_r w_r]^{R+1}$ which is closed due to the continuity of $g(\,\cdot\,)$. Observe that this set is the graph of the function $\rho(\,\cdot\,)$. We need to show that for any $u_n \to u^*$, we have $\rho(u_n) \to \rho(u^*)$. This is equivalent to showing that for any sequence $\{\rho(u_n)\}$, every subsequence $\{\rho(u_{n_k})\}$ satisfies $\rho(u_{n_k}) \to \rho(u^*)$. Let $u_n \to u^*$ and let $\{u_{n_k}\}$ be an arbitrary subsequence. Then, since $\mathcal{A}$ is compact, $(\rho(u_{n_k}), u_{n_k})$ has a convergent subsequence $(\rho(u_{n_{k_i}}), u_{n_{k_i}})$ with limit $(\rho^*, u^*) \in \mathcal{A}$. Moreover, $\rho(u^*) = \rho^*$ since $\rho(u^*)$ is the unique zero corresponding to u^*. Therefore, $\rho(u_{n_{k_i}}) \to \rho(u^*)$ and hence $\rho(\,\cdot\,)$ is continuous. Finally, continuity of ρ implies continuity of Q. An analogous argument can be made for κ^i and t^i_r. $\square$

We start with the proof of Proposition 5.1. The proofs of Propositions C.1 and C.2 then work relatively similarly.

Proof of Proposition 5.1. Fix a firm i and a vector $a^{-i} := (a^j)_{j \neq i}$ of production costs of i's competitors. Observe first that if $q^i_{tot}(a^i, a^{-i}) = 0$ for some $a^i > 0$, then also $q^i_{tot}(\tilde{a}^i, a^{-i}) = 0$ for any $\tilde{a}^i > a^i$. Indeed, if production costs a^i are too high to sell profitably, then it cannot be profitable to sell when production costs are $\tilde{a}^i > a^i$. Since $q^i_{tot}(\max_r w_r, a^{-i}) = 0$, it follows that there exists a cutoff $\bar{a}^i < \infty$ such that $q^i_{tot}(a^i, a^{-i}) = 0$ if and only if $a^i \geq \bar{a}^i$. Let $\underline{a}^i = \sup\bigl\{a^i \geq 0 \bigm| q^i_{tot}(a^i, a^{-i}) = \kappa^i\bigr\}$ with the convention that $\underline{a}^i = 0$ if $q^i_{tot}(a^i, a^{-i}) < \kappa^i$ for all $a^i \geq 0$. It follows from the definition of $\bar{a}^i$ that $\underline{a}^i \leq \bar{a}^i < \infty$. We will show that statements (i)–(iii) hold true for these choices of $\underline{a}^i$ and $\bar{a}^i$.

Statement (i). Suppose that $\underline{a}^i > 0$ as otherwise, there is nothing to show. Because $q^i_{tot}(a^i, a^{-i})$ is continuous in a^i by Lemma C.3, it follows that $q^i_{tot}(\underline{a}^i, a^{-i}) = \kappa^i$. Since firm i can profitably sell its entire capacity at $\underline{a}^i$, it can profitably sell its entire capacity also at lower production costs. This shows that $q^i_{tot}(a^i, a^{-i}) = \kappa^i$ for all $a^i < \underline{a}^i$. Since $Q(\underline{a}^i, a^{-i})$ is an equilibrium allocation, in which i sells its entire capacity, no firm has a profitable deviation also when production costs are equal to (a^i, a^{-i}) for $a^i < \underline{a}^i$. It follows that $Q(a^i, a^{-i}) = Q(\underline{a}^i, a^{-i})$ for all $a^i < \underline{a}^i$ by uniqueness. Finally, since the allocation is constant on $[0, \underline{a}^i)$, so are the prices.

Statement (ii). Because $Q(a^i, a^{-i})$ and hence also $p(a^i, a^{-i})$ are continuous in a^i by Lemma C.3, it is sufficient to show the statement on the open interval $(\underline{a}^i, \bar{a}^i)$. It follows from the definitions of $\underline{a}^i$ and $\bar{a}^i$ that $0 < q^i_{tot}(\,\cdot\,, a^{-i}) < \kappa^i$ on $(\underline{a}^i, \bar{a}^i)$ and hence $\rho^i(\,\cdot\,, a^{-i}) = 0$ on the entire interval. Since Φ^j is non-decreasing in a^i and ρ^j for any $j \neq i$, Theorem 3 by Milgrom and Roberts (1994) applies and establishes that $\rho^j(a)$ is non-decreasing in a^i. It follows similarly as in the proof of Lemma B.4.(i) that $q_r^{tot}(a)$ is non-increasing in a^i for any region r. It is an immediate consequence that $p_r(a)$ and $p_r'(a)$ are non-decreasing in a^i for any region r. Continuity and monotonicity imply that $q_r^{tot}(a)$ is differentiable for almost every a^i. Consider first a region r in $\Gamma_i^0(a)$ and denote by $\mathcal{I}_r(a) := \{i \mid q_r^i(a) > 0\}$ the set of all firms that sell a positive quantity in region r. Summing (1) over all $j \in \mathcal{I}_r(a)$, we get

$$\sum_{j \in \mathcal{I}_r(a)} \rho^j(a) = p_r'(q_r^{tot})q_r^{tot} + |\mathcal{I}_r(a)| p_r(q_r^{tot}) - \sum_{j \in \mathcal{I}_r(a)} (t_r^j + a^j). \tag{11}$$

Since $\rho^i(a) = 0$ for $a^i \in (\underline{a}^i, \bar{a}^i)$ and $\rho^j(a)$ for $j \neq i$ is non-decreasing in a^i, taking the derivative with respect to a^i at a differentiability point yields

$$0 \leq \sum_{j \in \mathcal{I}_r(a)} \frac{\partial \rho^j(a)}{\partial a^i} = \big(p_r''(q_r^{tot})q_r^{tot} + \big(|\mathcal{I}_r(a)| + 1\big)p_r'(q_r^{tot})\big)\frac{\partial q_r^{tot}}{\partial a^i} - 1.$$

Since $p_r'(q_r^{tot}) < 0$ and $p_r''(q_r^{tot}) \leq 0$, we obtain

$$\frac{\partial q_r^{tot}}{\partial a^i} \leq \frac{1}{p_r''(q_r^{tot})q_r^{tot} + \big(|\mathcal{I}_r(a)| + 1\big)p_r'(q_r^{tot})} < 0.$$

Together with continuity, this shows that q_r^{tot} is strictly decreasing in a^i for every region $r \in \Gamma_i^0(a)$. It is an immediate consequence that $p_r(a)$ is strictly increasing in a^i for regions $r \in \Gamma_i^0(a)$.

Suppose now that prices $p_r(a)$ are strictly increasing in a^i for regions in $\Gamma_i^{k-1}(a)$. We will show that then prices have to be strictly increasing in regions in $\Gamma_i^k(a)$ as well. Fix a firm $j \in \Gamma_i^k(a) \setminus \Gamma_i^{k-1}(a)$ with $\rho^j(a) > 0$ and let $R_j(a) := \{r \mid q_r^j(a) > 0\}$ denote the regions, to which j is selling a positive quantity. Solving (1) for firm j and summing over all regions in $R_j(a)$, we obtain

$$\kappa^j = \sum_{r \in R_j(a)} q_r^j(a) = \sum_{r \in R_j(a)} \frac{\rho^j(a) + t_r^j + a^j - p_r(a)}{p_r'(a)}. \tag{12}$$

Since $\rho^j(a)$ is continuous in a^i by Lemma C.3, $\rho^j(a) > 0$ in a neighborhood of a^i and hence

$q^j_{tot}(a) = \kappa^j$ in a neighborhood. Taking the derivative of (12) at a differentiability point yields

$$0 = \sum_{r \in R_j(a)} \frac{\frac{\partial \rho^j(a)}{\partial a^i} - \frac{\partial p_r(a)}{\partial a^i} - \frac{\partial p'_r(a)}{\partial a^i} q^j_r(a)}{p'_r(a)}.$$

Solving for $\frac{\partial \rho^j(a)}{\partial a^i}$, we obtain

$$\frac{\partial \rho^j(a)}{\partial a^i} = \frac{1}{\sum_{r \in R_j(a)} \frac{1}{p'_r(a)}} \sum_{r \in R_j(a)} \frac{\frac{\partial p_r(a)}{\partial a^i} + q^j_r \frac{\partial p'_r(a)}{\partial a^i}}{p'_r(a)} > 0, \tag{13}$$

where we have used the fact that $R_j(a) \cap \Gamma_i^{k-1}(a) \neq \emptyset$, prices $p_r(a)$ are strictly increasing for $r \in R_j(a) \cap \Gamma_i^{k-1}(a)$, and that $p_r(a)$ and $p'_r(a)$ are non-decreasing for every region r. Fix now a region $r \in \Gamma_i^k(a) \setminus \Gamma_i^{k-1}(a)$. Taking the partial derivative of (11) with respect to a^i and solving for the marginal change in q_r^{tot}, we obtain

$$\frac{\partial q_r^{tot}}{\partial a^i} = \frac{\sum_{j \in \mathcal{I}_r(a)} \frac{\partial \rho^j(a)}{\partial a^i}}{p''_r(q_r^{tot}) q_r^{tot} + \big(|\mathcal{I}_r(a)| + 1\big) p'_r(q_r^{tot})} < 0,$$

where we have used that $\mathcal{I}_r(a) \cap \Gamma_i^k(a) \neq \emptyset$ and hence the numerator is strictly positive by (13). This concludes the proof that $p_r(a)$ is strictly increasing in a^i for any region r in $\Gamma_i^k(a)$. Thus, by induction, $p_r(a)$ is strictly increasing in a^i for any $r \in \Gamma_i(a)$.

Consider now the restriction $Q(a)|_{\Gamma_i(a)^c}$ of $Q(a)$ to regions in $\big(\Gamma_i(a)\big)^c$ and to firms selling to those regions. By definition of $\Gamma_i(a)$, those firms have either excess capacity or those firms do not sell to regions in $\Gamma_i(a)$. For firms j with excess capacity, it follows that $\rho^j(a) = 0$ in a neighborhood of a^i. Thus, by Lemma 4.1, $Q(a)|_{\Gamma_i(a)^c}$ satisfies (2) in a neighborhood of a^i. Therefore, $Q(a)|_{\Gamma_i(a)^c}$ is locally constant by uniqueness. It follows that $q_r^{tot}(a)$ is locally constant for regions $r \notin \Gamma_i(a)$ and hence $p_r(a)$ is locally constant as well.

Finally, fix a firm $j \in \Gamma_i(a) \setminus \{i\}$ that is not producing at capacity. Since $\rho^j(a)$ is locally constant and $p_r(a)$ is strictly increasing for $r \in \Gamma_i(a)$, taking the partial derivative in (1) shows that $q^j_r(a)$ is strictly increasing in a^i.

Statement (iii). Similarly to (i), since $Q(\bar{a}^i, a^{-i})$ is an equilibrium allocation, in which i sells nothing, no firm has a profitable deviation also for production costs (a^i, a^{-i}) for $a^i > \bar{a}^i$. $\square$

Proof of Proposition C.1. Fix a firm i and a vector $\kappa^{-i} := (\kappa^j)_{j\neq i}$ of production capacities of i's competitors. Let $\underline{\kappa}^i := \lim_{k\to\infty} q^i_{tot}(k, \kappa^{-i})$ denote the equilibrium quantity produced by firm i if it were not capacity constrained. We will show statements (i) and (ii) hold for this choice of $\underline{\kappa}^i$.

Statement (i). Since $\underline{\kappa}^i$ is the equilibrium quantity produced by firm i if it were not capacity constrained, the firm has no profitable use for additional capacity beyond $\underline{\kappa}^i$. This implies that firm i would not change its allocation for any $\kappa^i > \underline{\kappa}^i$ and hence $Q(\kappa^i, \kappa^{-i}) = Q(\underline{\kappa}^i, \kappa^{-i})$ by uniqueness. Since the allocation is constant on the interval $(\underline{\kappa}^i, \infty)$, so are the prices.

Statement (ii). Suppose $\underline{\kappa}^i > 0$ as otherwise there is nothing to show. Because $Q(\kappa^i, \kappa^{-i})$ and hence prices $p(\kappa^i, \kappa^{-i})$ are continuous in κ^i by Lemma C.3, it is sufficient to show the statement for the open interval $(0, \underline{\kappa}^i)$. It follows from the definition of $\underline{\kappa}^i$ that $q^i_{tot}(\kappa^i, \kappa^{-i}) = \kappa^i$ for all $\kappa^i < \underline{\kappa}^i$. Since Φ^j is non-increasing in κ^i for any firm j, it follows that Φ is monotone. Therefore, Theorem 3 by Milgrom and Roberts (1994) applies and establishes that $\rho^j(\kappa)$ is non-increasing in κ^i.[21] It follows from Lemma B.4.(i) that $q^{tot}_r(\kappa)$ is non-decreasing in κ^i for any region r. It is an immediate consequence that $p_r(\kappa)$ and $p'_r(\kappa)$ are non-increasing in κ^i for any region r. Continuity and monotonicity imply that $q^{tot}_r(\kappa)$ is differentiable for almost every κ^i. Solving (1) for firm i and summing over all regions $r \in \Gamma^0_i(\kappa)$, we obtain

$$\kappa^i = \sum_{r\in\Gamma^0_i(\kappa)} q^i_r(\kappa) = \sum_{r\in\Gamma^0_i(\kappa)} \frac{\rho^i(\kappa) + t^i_r + a^i - p_r(\kappa)}{p'_r(\kappa)}. \tag{14}$$

Taking the derivative of (14) at a differentiability point yields

$$1 = \sum_{r\in\Gamma^0_i(\kappa)} \frac{\frac{\partial\rho^i(\kappa)}{\partial\kappa^i} - \frac{\partial p_r(\kappa)}{\partial\kappa^i} - \frac{\partial p'_r(\kappa)}{\partial\kappa^i} q^i_r(\kappa)}{p'_r(\kappa)}.$$

Since $p_r(\kappa)$ and $p'_r(\kappa)$ are non-increasing in κ^i and $p'_r(\kappa) < 0$, we obtain

$$\frac{\partial\rho^i(\kappa)}{\partial\kappa^i} = \frac{1}{\sum_{r\in\Gamma^0_i(\kappa)} \frac{1}{p'_r(\kappa)}} \left[1 + \sum_{r\in\Gamma^0_i(\kappa)} \frac{\frac{\partial p_r(\kappa)}{\partial\kappa^i} + q^i_r \frac{\partial p'_r(\kappa)}{\partial\kappa^i}}{p'_r(\kappa)}\right] < 0. \tag{15}$$

Fix now a region $r \in \Gamma^0_i(\kappa)$. Summing (1) over all $j \in \mathcal{I}_r(\kappa)$, where $\mathcal{I}_r(\kappa)$ is the set of firms that

[21] In order to directly apply the theorem, define $\tilde{\Phi}$ by replacing κ^i in Φ with $-\kappa^i$. Observe now that $\tilde{\Phi}^j$ non-decreasing in $-\kappa^i$ and Φ is monotone. Therefore, $\rho^j(\kappa)$ is non-decreasing in $-\kappa^i$ and hence non-increasing in κ^i.

are active in region r at equilibrium when vector of production capacities is κ, we get

$$\sum_{j \in \mathcal{I}_r(\kappa)} \rho^j(\kappa) = p'_r(q_r^{tot})q_r^{tot} + |\mathcal{I}_r(a)|p_r(q_r^{tot}) - \sum_{j \in \mathcal{I}_r(\kappa)} (t_r^j + a^j).$$

Taking the derivative with respect to κ^i at a differentiability point yields

$$\sum_{j \in \mathcal{I}_r(\kappa)} \frac{\partial \rho^j(\kappa)}{\partial \kappa^i} = \big(p''_r(q_r^{tot})q_r^{tot} + \big(|\mathcal{I}_r(\kappa)| + 1\big)p'_r(q_r^{tot})\big)\frac{\partial q_r^{tot}}{\partial \kappa^i}.$$

Solving for $\frac{\partial q_r^{tot}}{\partial \kappa^i}$, we obtain

$$\frac{\partial q_r^{tot}}{\partial \kappa^i} = \frac{\sum_{j \in \mathcal{I}_r(\kappa)} \frac{\partial \rho^j(\kappa)}{\partial \kappa^i}}{p''_r(q_r^{tot})q_r^{tot} + \big(|\mathcal{I}_r(\kappa)| + 1\big)p'_r(q_r^{tot})} > 0$$

where we have used the fact that the numerator is strictly negative by (15). Together with continuity, this shows that q_r^{tot} is strictly decreasing in κ^i for every region $r \in \Gamma_i^0(\kappa)$. An induction argument analogous to the one used in Proposition 5.1.(ii) shows that $p_r(\kappa)$ is strictly decreasing in κ^i for any $r \in \Gamma_i(\kappa)$. Finally, arguments similar to the one used in Proposition 5.1.(ii) show that $p_r(\kappa)$ is constant in κ^i for any $r \notin \Gamma_i(\kappa)$ and $q_{tot}^j(\kappa)$ is non-increasing in κ^i for any $j \neq i$.

□

Proof of Proposition C.2. Fix a firm i and a region r and a matrix T_{-r}^{-i} of transportation costs without the element t_r^i. Observe first that if $q_r^i(t_r^i, T_{-r}^{-i}) = 0$ for some $t_r^i \geq 0$, then also $q_r^i(\tilde{t}_r^i, T_{-r}^{-i}) = 0$ for any $\tilde{t}_r^i > t_r^i$. Indeed, if the transportation costs t_r^i are too high to sell profitably to region r, then it cannot be profitable to sell to region r when transportation costs are $\tilde{t}_r^i > t_r^i$. Since $q_r^i(w_r, T_{-r}^{-i}) = 0$, it follows that there exists a cutoff $\bar{t}_r^i < \infty$ such that $q_{tot}^i(t_r^i, T_{-r}^{-i}) = 0$ if and only if $t_r^i \geq \bar{t}_r^i$. Let $\underline{t}_r^i = \sup\big\{t_r^i \geq 0 \;\big|\; q_{tot}^i(t_r^i, T_{-r}^{-i}) = \kappa^i\big\}$ with the convention that $\underline{t}_r^i = 0$ if $q_{tot}^i(t_r^i, T_{-r}^{-i}) < \kappa^i$ for all $t_r^i \geq 0$. It follows from the definition of $\bar{t}_r^i$ that $\underline{t}_r^i \leq \bar{t}_r^i < \infty$. We will show that statements (i)–(iii) hold true for these choices of $\underline{t}_r^i$ and $\bar{t}_r^i$.

Statement (i). Suppose that $\underline{t}_r^i > 0$ as otherwise, there is nothing to show. Because $q_{tot}^i(t_r^i, T_{-r}^{-i})$ is continuous in t_r^i by Lemma C.3, it follows that $q_{tot}^i(\underline{t}_r^i, T_{-r}^{-i}) = \kappa^i$. Since firm i can profitably sell its entire capacity at $\underline{t}_r^i$, it can profitably sell its entire capacity also at lower transportation costs. This shows that $q_{tot}^i(t_r^i, T_{-r}^{-i}) = \kappa^i$ for all $t_r^i < \underline{t}_r^i$.

Statement (ii). Because $Q(t_r^i, T_{-r}^{-i})$ and hence also $p(t_r^i, T_{-r}^{-i})$ are continuous in t_r^i by Lemma C.3, it is sufficient to show the statement on the open interval $(\underline{t}_r^i, \bar{t}_r^i)$. It follows from the definitions of $\underline{t}_r^i$ and $\bar{t}_r^i$ that $0 < q_{tot}^i(\,\cdot\,, T_{-r}^{-i}) < \kappa^i$ on $(\underline{t}_r^i, \bar{t}_r^i)$ and hence $\rho^i(\,\cdot\,, T_{-r}^{-i}) = 0$ on the entire interval. Since Φ^j is non-decreasing in t_r^i and ρ^j for any $j \neq i$, Theorem 3 by Milgrom and Roberts (1994) applies and establishes that $\rho^j(a)$ is non-decreasing in t_r^i. It follows similarly as in the proof of Lemma B.4.(i) that $q_{r'}^{tot}(T)$ is non-increasing in t_r^i for any region r'. It is an immediate consequence that $p_{r'}(T)$ and $p'_{r'}(T)$ are non-decreasing in t_r^i for any region r'. The remainder of the argument works analogously to the proof of Proposition 5.1.(ii).

Statement (iii). Since $q_r^i(\bar{t}_r^i, T_{-r}^{-i})$ is an equilibrium allocation, in which firm i sells nothing to region r, firm i cannot profitably sell a positive amount to region r when when transportation costs are $t_r^i > \bar{t}^i$. This implies that firm i would not change its allocation for any $t_r^i > \bar{t}_r^i$, and hence $Q(t_r^i, T_{-r}^{-i}) = Q(\bar{t}^i, T_{-r}^{-i})$ by uniqueness. Since the allocation is constant on the interval $[\bar{t}_r^i, \infty)$, so are the prices.

□

Proof of Proposition 5.2. Fix a firm i. By Proposition 5.1, the vector of equilibrium prices $p(a)$ is non-decreasing in a^i. This implies that the total quantity sold in each region r, q_r^{tot}, is non-increasing in a^i because the prices are strictly decreasing in quantities. The result follows by observing that the derivative of the aggregate consumer surplus with respect to q_r^{tot} is $-p'_r(q_r^{tot})q_r^{tot}$, and thus positive for any decreasing inverse demand function. The proof of the remaining parts follows from an analogous reasoning by using Propositions C.1 and C.2 instead of Proposition 5.1. □

Proof of Proposition 5.3. We use $p(q) = p_r(q)$ to denote the price function in all regions since regional demand functions are homogeneous. By Lemma 4.1, there exists $\rho = (\rho^1, \ldots, \rho^N)$ such that $\rho^i = p'(q_r^{tot})q_r^i + p(q_r^{tot}) - t_r^i - a^i$ for every firm i and every region r. Because demand functions are linear, which implies that inverse demand functions are also linear, plugging $p(x) = w - \alpha x$ into the previous equation we obtain

$$\rho^i = -\alpha q_r^i + w - \alpha q_r^{tot} - t_r^i - a^i \tag{16}$$

for any firm i and any region r. Summing (16) over all regions and dividing by R yields

$$\rho^i = w - a^i - \frac{1}{R}\left(\alpha\kappa^i + \alpha\sum_j \kappa^j + \sum_r t_r^i\right), \tag{17}$$

where we have used that all firms are producing at capacity and hence $\sum_r q_r^j = \kappa^j$ for any firm j. Equating (16) with (17) and solving for all components of q_r, we obtain

$$2q_r^i + \sum_{j\neq i} q_r^j = \frac{1}{R}\left(\kappa^i + \sum_j \kappa^j\right) + \frac{1}{\alpha}A_r^i(T) \qquad \forall\, i, r$$

where we recall that $A_r^i(T) := \frac{1}{R}\sum_{r'} t_{r'}^i - t_r^i$ was defined in Section 5.2. This system of linear equations admits the solution

$$q_r^i = \frac{\kappa^i}{R} + \frac{1}{\alpha}\left(A_r^i(T) - \frac{A_r(T)}{N+1}\right) \qquad \forall\, i, r. \tag{18}$$

Summing (18) over all firms, we obtain

$$q_r^{tot} = \frac{1}{R}\sum_{i=1}^{N}\kappa^i + \frac{A_r(T)}{\alpha(N+1)} \qquad \forall\, r. \tag{19}$$

By the linearity of the inverse demand function, i.e., $p(x) = w - \alpha x$,

$$CS(T) = \sum_r CS_r(T) = \sum_r \frac{\alpha(q_r^{tot})^2}{2} = \frac{\alpha}{2R}\left(\sum_{i=1}^{N}\kappa^i\right)^2 + \frac{1}{2\alpha(N+1)^2}\sum_r A_r^2(T), \tag{20}$$

where we have used that $\sum_r A_r(T) = 0$, by definition. Because the function above is convex and symmetric in $A(T)$, it is Schur-convex. We recall that a function $f : R^d \to R$ is said to be Schur-convex if it preserves the majorization order (**?**), i.e., a majorizes b implies $f(a) \geq f(b)$. Therefore, if $A(T)$ majorizes $A(\tilde{T})$, then $CS(T) \geq CS(\tilde{T})$. This concludes the proof of the second statement. To prove the first statement, we omit the dependence of $A(T)$ on T for the sake of brevity. Taking the partial derivative of (20) with respect to t_r^i, we obtain

$$\frac{\partial CS}{\partial t_r^i} = \frac{1}{\alpha(N+1)^2}\sum_{r'} A_{r'}\frac{\partial A_{r'}}{\partial t_r^i} = \frac{1}{\alpha(N+1)^2}\left(\frac{1}{R}\sum_{r'} A_{r'} - A_r\right) = -\frac{A_r}{(N+1)^2\alpha}, \tag{21}$$

where we have used the definition of A_r in the second equality, and the fact that $\sum_{r'} A_{r'} = 0$ in the last equality. The proof of the first statement follows from the fact that $\alpha > 0$. $\square$

Proof of Proposition 5.4. By linearity of the demand function, which in turn implies the linearity of the inverse demand function $p(x) = w - \alpha x$, firm i's profit takes the form

$$\pi^i = \sum_r (w - \alpha q_r^{tot} - t_r^i) q_r^i - a^i \kappa^i, \tag{22}$$

where we have used that firm i is producing at capacity and hence $q_{tot}^i = \kappa^i$. Since every firm is assumed to produce at capacity, the equilibrium allocation $Q = \left(q_r^i\right)_{\substack{i=1,\dots,N\\ r=1,\dots,R}}$ is given by (18). Observe that

$$\frac{\partial A_{r'}^j}{\partial t_r^i} = \left(\frac{1}{R} - 1_{\{r'=r\}}\right) 1_{\{j=i\}}. \tag{23}$$

Therefore, (18) and (19) imply that

$$\frac{\partial q_{r'}^i}{\partial t_r^i} = \frac{N}{\alpha(N+1)}\left(\frac{1}{R} - 1_{\{r'=r\}}\right), \qquad \frac{\partial q_{r'}^{tot}}{\partial t_r^i} = \frac{1}{\alpha(N+1)}\left(\frac{1}{R} - 1_{\{r'=r\}}\right).$$

Taking the partial derivative in (22) with respect to t_r^i yields

$$\begin{aligned}
\frac{\partial \pi^i}{\partial t_r^i} &= \frac{N}{\alpha(N+1)} \sum_{r'} (w - \alpha q_{r'}^{tot} - t_{r'}^i)\left(\frac{1}{R} - 1_{\{r'=r\}}\right) - \frac{1}{N+1} \sum_{r'} q_{r'}^i \left(\frac{1}{R} - 1_{\{r'=r\}}\right) - q_r^i \\
&= \frac{N}{(N+1)}\left(q_r^{tot} - \frac{1}{R}\sum_{r'} q_{r'}^{tot} - \frac{1}{\alpha} A_r^i - \frac{\kappa^i}{RN} - q_r^i\right) \\
&= \frac{2N}{\alpha(N+1)}\left(\frac{A_r}{N+1} - A_r^i - \frac{\alpha(N+1)\kappa^i}{2RN}\right) \qquad (24) \\
&= \frac{2N^2}{\alpha(N+1)^2}\left(\frac{1}{N}\sum_{j\neq i} A_r^j - A_r^i - \frac{\alpha(N+1)^2\kappa^i}{2RN^2}\right).
\end{aligned}$$

The first statement thus follows if we choose $\theta := \frac{\alpha(N+1)^2}{2RN^2}$. For the second statement, observe that (18) and (23) imply

$$\frac{\partial q_{r'}^j}{\partial t_r^i} = -\frac{1}{\alpha(N+1)}\left(\frac{1}{R} - 1_{\{r'=r\}}\right).$$

Following similar steps to what done above, we obtain

$$\frac{\partial \pi^j}{\partial t_r^i} = \frac{2}{\alpha(N+1)}\left(A_r^j - \frac{A_r}{N+1}\right) = \frac{2N}{\alpha(N+1)^2}\left(A_r^j - \frac{1}{N}\sum_{k\neq j} A_r^k\right) \tag{25}$$

for any $j \neq i$, thereby proving the second statement. It is easy to verify, using an analogous argument, that (24) and (25) remain the same when a subset of the rival firms are not in $\mathcal{C}(T)$.

□

Proof of Corollary 5.5. Since A_r^j is constant and A_r^i is decreasing in t_r^i, it follows from Statements (i) and (ii) of Proposition 5.4 that there exist $\hat{t}_r^{ij}$ and $\hat{t}_r^{ii}$ such that $A_r^j > \sum_{k\neq j} A_r^k$ if and only if $t_r^i > \hat{t}_r^{ij}$ and $A_r^i < \sum_{j\neq i} A_r^j - \theta\kappa^i$ if and only if $t_r^i > \hat{t}_r^{ii}$. The result follows for $\underline{t}_r^i := \min_j \hat{t}_r^{ij}$ and $\bar{t}_r^i := \max_j \hat{t}_r^{ij}$. □

Proof of Proposition 5.6. Summing (25) for every $j \neq i$ and (24) yields

$$\frac{\partial \Pi}{\partial t_r^i} = \frac{2}{\alpha(N+1)}\left(\frac{N+2}{N+1}A_r - (N+1)A_r^i\right) - \frac{\kappa^i}{R}. \tag{26}$$

Summing (21) and (26), we obtain

$$\begin{aligned}\frac{\partial W}{\partial t_r^i} &= \frac{1}{\alpha(N+1)^2}\big((2N+3)A_r - 2(N+1)^2 A_r^i\big) - \frac{\kappa^i}{R}\\ &= \frac{1}{\alpha(N+1)^2}\left((2N+3)\sum_{j\neq i} A_r^j - (2N^2+2N-1)A_r^i\right) - \frac{\kappa^i}{R}.\end{aligned} \tag{27}$$

Because, by (23), A_r^i is decreasing in t_r^i and A_r^j, $j \neq i$, is constant in t_r^i, this implies that there exists a threshold above which $\frac{\partial W}{\partial t_r^i} > 0$. This proves the first statement. To prove the second statement, observe that adding (20) and (22) for all firms $j = 1, 2, \ldots, N$, and differentiating with respect to κ^i and a^i yields

$$\frac{\partial W}{\partial \kappa^i} = \rho^i + \frac{\alpha\kappa^i}{R} > 0 \quad \text{and}$$

$$\frac{\partial W}{\partial a^i} = -\kappa^i < 0,$$

respectively. □

D Data

In this section, we provide the data used to calibrate the model in Section 6. Production capacities are reported in Tables 6 and 7, production costs are given in Tables 8 and 9, regional consumption data in Tables 10 and 11, and willingness to pay parameters in Tables 12 and 13. We provide values of the shipping costs (SH_r^i) and tax rates (tx_r^i) in Tables 14 and 15. The transportation costs are computed using the formula $T_t^{i,r} = SH_r^i + tx_r^i(a_t^i + SH_r^i)$.

κ_t^i	2012Q1	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
Firm 1	357	357	357	357	368	376	384	407
Firm 2	319	319	319	319	389	389	389	389
Firm 3	213	213	213	213	259	259	259	259
Firm 4	1334	1334	1334	1334	1296	1296	1296	1296
Firm 5	488	488	488	488	488	488	488	488
Firm 6	270	270	270	270	270	270	270	270
Firm 7	150	150	150	150	183	183	183	183
Firm 8	75	75	75	75	75	75	75	75
Firm 9	625	625	625	625	625	625	625	625
Firm 10	319	319	319	319	389	389	389	389
Firm 11	213	213	213	213	259	259	259	259
Firm 12	319	319	319	319	389	389	389	389
Firm 13	213	213	213	213	259	259	259	259

Table 6: Production capacities κ_t^i (in kiloton) for the period 2012Q1–2013Q4

κ_t^i	2014Q1	2014Q2	2014Q3	2014Q4	2015Q1	2015Q2	2015Q3	2015Q4	2016Q1
Firm 1	424	448	464	492	512	536	560	584	600
Firm 2	361	361	361	361	369	369	369	369	532
Firm 3	240	240	240	240	246	246	246	246	243
Firm 4	1296	1296	1296	1296	1062	1062	1062	1062	1015
Firm 5	488	488	488	488	488	488	488	488	488
Firm 6	270	270	270	270	270	270	270	270	270
Firm 7	183	183	183	183	183	183	183	183	212
Firm 8	75	75	75	75	78	78	78	78	91
Firm 9	625	625	625	625	625	625	625	625	625
Firm 10	361	361	361	361	369	369	369	369	532
Firm 11	240	240	240	240	246	246	246	246	243
Firm 12	361	361	361	361	369	369	369	369	532
Firm 13	240	240	240	240	246	246	246	246	243

Table 7: Production capacities κ_t^i (in kiloton) for the period 2014Q1–2016Q1

a_t^i	2012Q1	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
Firm 1	295	316	339	336	316	300	256	256
Firm 2	315	335	358	355	335	319	275	276
Firm 3	351	372	395	392	372	356	312	312
Firm 4	287	308	331	328	308	292	248	248
Firm 5	267	288	311	308	288	272	228	228
Firm 6	247	268	291	288	268	252	208	208
Firm 7	311	332	355	352	332	316	272	272
Firm 8	327	348	371	368	348	332	288	288
Firm 9	327	348	371	368	348	332	288	288
Firm 10	315	335	358	355	335	319	275	276
Firm 11	351	372	395	392	372	356	312	312
Firm 12	315	335	358	355	335	319	275	276
Firm 13	351	372	395	392	372	356	312	312

Table 8: Production cost a_t^i (in \$/ton) for the period 2012Q1–2013Q4

a_t^i	2014Q1	2014Q2	2014Q3	2014Q4	2015Q1	2015Q2	2015Q3	2015Q4	2016Q1
Firm 1	282	299	300	311	294	279	278	259	248
Firm 2	302	318	319	330	313	298	297	279	267
Firm 3	338	355	356	367	350	335	334	315	304
Firm 4	274	291	292	303	286	271	270	251	240
Firm 5	254	271	272	283	266	251	250	231	220
Firm 6	234	251	252	263	246	231	230	211	200
Firm 7	298	315	316	327	310	295	294	275	264
Firm 8	314	331	332	343	326	311	310	291	280
Firm 9	314	331	332	343	326	311	310	291	280
Firm 10	302	318	319	330	313	298	297	279	267
Firm 11	338	355	356	367	350	335	334	315	304
Firm 12	302	318	319	330	313	298	297	279	267
Firm 13	338	355	356	367	350	335	334	315	304

Table 9: Production cost a_t^i (in \$/ton) for the period 2014Q1–2016Q1

$C_{r,t}$	2012Q1	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
N. America	186	87	131	268	258	137	155	180
S. America	425	986	460	578	549	859	599	655
Europe	322	339	287	350	440	286	395	558
Africa	170	81	117	103	114	90	61	165
Asia	511	519	1,716	1,004	352	631	1,165	1,101
Oceania	156	33	16	141	134	28	31	130

Table 10: Regional consumption $C_{r,t}$ (in kiloton) for the period 2012Q1–2013Q4

$C_{r,t}$	2014Q1	2014Q2	2014Q3	2014Q4	2015Q1	2015Q2	2015Q3	2015Q4	2016Q1
N. America	193	148	233	289	300	93	233	229	312
S. America	669	907	898	559	543	1,008	657	462	510
Europe	351	263	282	306	407	296	363	341	416
Africa	162	55	92	145	99	131	141	103	188
Asia	468	772	1,201	1,268	607	1,268	1,546	1,283	416
Oceania	222	59	51	88	165	101	35	151	173

Table 11: Regional consumption $C_{r,t}$ (in kiloton) for the period 2014Q1–2016Q1

$w_{r,t}$	2012Q1	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
N. America	432.19	454.24	476.30	485.12	418.96	423.37	374.86	339.58
S. America	440.00	512.00	489.60	468.00	440.00	440.00	408.00	348.00
Europe	504.00	500.00	500.00	476.00	452.00	452.00	440.00	348.00
Africa	460.00	476.00	480.00	476.00	432.00	424.00	412.00	344.00
Asia	541.60	464.00	464.00	464.00	452.00	416.00	406.40	332.00
Oceania	549.60	472.00	472.00	472.00	460.00	424.00	414.40	300.00

Table 12: Regional willingness to pay $w_{r,t}$ (in \$/ton) for the period 2012Q1–2013Q4

$w_{r,t}$	2014Q1	2014Q2	2014Q3	2014Q4	2015Q1	2015Q2	2015Q3	2015Q4	2016Q1
N. America	441.01	454.24	401.32	388.09	401.32	376.63	386.33	380.15	313.12
S. America	420.00	420.00	432.00	400.00	409.60	400.00	396.00	372.00	300.00
Europe	460.00	456.00	432.00	432.00	436.00	416.00	416.00	404.00	356.00
Africa	440.00	440.00	432.00	430.40	428.00	412.00	412.00	396.00	360.00
Asia	341.60	370.40	404.00	404.00	392.00	388.80	385.60	369.60	326.40
Oceania	400.00	400.00	392.00	392.00	384.00	384.00	380.00	368.00	368.00

Table 13: Regional willingness to pay $w_{r,t}$ (in \$/ton) for the period 2014Q1–2016Q1

SH_r^i	N. America	S. America	Europe	Africa	Asia	Oceania
Firm 1	8	10	10	20	40	40
Firm 2	28	36	28	20	4	10
Firm 3	28	36	28	20	4	10
Firm 4	5	12	12	24	20	24
Firm 5	16	16	12	24	28	32
Firm 6	20	12	24	12	12	20
Firm 7	20	16	12	20	24	28
Firm 8	20	20	20	12	12	24
Firm 9	8	12	12	24	20	24
Firm 10	28	36	28	20	4	10
Firm 11	28	36	28	20	4	10
Firm 12	28	36	28	20	4	10
Firm 13	28	36	28	20	4	10

Table 14: Shipping cost SH_r^i (in \$/ton)

tx_r^i	N. America	S. America	Europe	Africa	Asia	Oceania
Firm 1	0%	0%	0%	0%	0%	0%
Firm 2	0%	0%	6%	0%	0%	0%
Firm 3	0%	0%	6%	0%	0%	0%
Firm 4	0%	0%	6%	0%	0%	0%
Firm 5	0%	0%	0%	0%	0%	0%
Firm 6	0%	0%	6%	0%	0%	0%
Firm 7	0%	0%	0%	0%	0%	0%
Firm 8	0%	0%	0%	0%	0%	0%
Firm 9	0%	0%	0%	0%	0%	0%
Firm 10	0%	0%	6%	0%	0%	0%
Firm 11	0%	0%	6%	0%	0%	0%
Firm 12	0%	0%	6%	0%	0%	0%
Firm 13	0%	0%	6%	0%	0%	0%

Table 15: Import taxes tx_r^i (in %)

E In-sample Results

Model	N. America	S. America	Europe	Africa	Asia	Oceania
2012Q1	399.38	417.52	463.79	421.42	485.66	484.85
2012Q2	421.30	476.29	466.94	439.72	434.79	438.32
2012Q3	444.67	465.44	473.83	449.60	441.53	446.20
2012Q4	448.77	448.83	454.95	445.98	440.31	444.73
2013Q1	396.77	422.11	430.45	409.26	424.79	428.52
2013Q2	394.66	418.16	426.34	398.89	394.43	399.09
2013Q3	348.67	383.58	406.38	376.49	376.57	379.99
2013Q4	324.87	337.97	337.31	331.03	322.10	297.26
2014Q1	400.01	398.49	427.49	402.70	334.27	377.21
2014Q2	414.44	402.84	429.11	408.40	360.66	382.36
2014Q3	381.33	412.48	412.66	404.32	386.67	377.96
2014Q4	374.58	390.37	415.05	406.20	389.18	381.01

Table 16: The model's in-sample predicted prices ($/ton) for the period 2012Q1–2014Q4.

Historical	N. America	S. America	Europe	Africa	Asia	Oceania
2012Q1	394.88	434.15	469.26	429.54	475.11	483.11
2012Q2	437.01	483.23	475.78	450.92	448.00	456.00
2012Q3	456.48	481.17	482.12	467.54	464.00	472.00
2012Q4	436.77	443.69	452.37	438.31	458.00	466.00
2013Q1	412.65	413.69	431.48	408.46	414.18	422.18
2013Q2	385.01	416.92	434.34	412.15	407.42	415.42
2013Q3	349.62	376.92	392.00	371.69	354.31	362.31
2013Q4	313.36	328.92	326.92	318.77	314.46	292.75
2014Q1	402.58	394.83	414.25	388.15	337.91	307.69
2014Q2	402.17	381.08	436.31	380.15	364.62	389.69
2014Q3	389.08	415.23	416.46	416.92	381.02	388.00
2014Q4	370.49	384.09	407.23	409.32	389.75	380.62

Table 17: The historical quarterly prices ($/ton) for the period 2012Q1–2014Q4.

	N. America	S. America	Europe	Africa	Asia	Oceania
2012Q1	1.14%	-3.83%	-1.17%	-1.89%	2.22%	0.36%
2012Q2	-3.59%	-1.44%	-1.86%	-2.49%	-2.95%	-3.88%
2012Q3	-2.59%	-3.27%	-1.72%	-3.84%	-4.84%	-5.47%
2012Q4	2.75%	1.16%	0.57%	1.75%	-3.86%	-4.56%
2013Q1	-3.85%	2.04%	-0.24%	0.20%	2.56%	1.50%
2013Q2	2.51%	0.30%	-1.84%	-3.22%	-3.19%	-3.93%
2013Q3	-0.27%	1.77%	3.67%	1.29%	6.28%	4.88%
2013Q4	3.67%	2.75%	3.18%	3.85%	2.43%	1.54%
2014Q1	-0.64%	0.93%	3.20%	3.75%	-1.08%	22.59%
2014Q2	3.05%	5.71%	-1.65%	7.43%	-1.08%	-1.88%
2014Q3	-1.99%	-0.66%	-0.91%	-3.02%	1.48%	-2.59%
2014Q4	1.10%	1.64%	1.92%	-0.76%	-0.15%	0.10%

Table 18: The relative error $\frac{\text{output price}-\text{historical price}}{\text{historical price}}$ for the period 2012Q1–2014Q4.

Elasticity	N. America	S. America	Europe	Africa	Asia	Oceania
$S_{r,t} = 0.1C_{r,t}$	0.040	0.032	0.027	0.048	0.027	0.037
$S_{r,t} = 0.3C_{r,t}$	0.042	0.033	0.028	0.049	0.028	0.038
$S_{r,t} = 0.5C_{r,t}$	0.043	0.034	0.028	0.051	0.029	0.039

Table 19: The regional elasticity parameters that yield the lowest price root mean square error (RMSE) for the period 2012Q1–2014Q4 for different values of $S_{r,t}$.

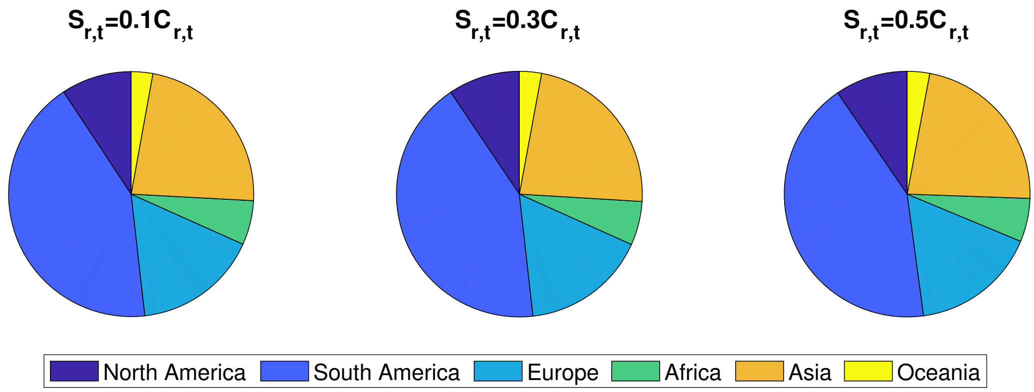

Figure 7: The charts show firm 1's model's predicted allocation of exports across regions over the entire period 2012Q1–2016Q1 for different values of $S_{r,t}$.

F Robustness

In this section, we analyze how the estimated output elasticity vector, out-of-sample prices, and firm 1's sales change if we pick different values for the storage capacity parameters $S_{r,t}$. It appears from Table 19 that there are small differences between the elasticity parameter estimated in Section 6 (middle row of the Table), and the corresponding estimates when we pick either $S_{r,t} = 0.1C_{r,t}$ or $S_{r,t} = 0.5C_{r,t}$. Out-of-sample predictions reported in Table 20 also highlight the small sensitivity of the results to little perturbations of the storage capacity parameter. Figure 7 shows that the allocation of firm 1 is not affected by the storage parameter $S_{r,t}$

$S_{r,t} = 0.1C_{r,t}$	N. America	S. America	Europe	Africa	Asia	Oceania
2015-Q1	377.87	393.74	412.64	398.75	374.87	368.98
2015-Q2	357.23	383.47	394.81	384.07	369.60	365.67
2015-Q3	364.40	380.20	394.94	383.65	367.38	363.08
2015-Q4	353.46	357.42	380.68	366.49	350.33	348.68
2016-Q1	301.23	296.61	339.93	337.60	314.01	343.36
$S_{r,t} = 0.3C_{r,t}$						
2015-Q1	378.33	393.78	412.97	398.72	375.29	369.56
2015-Q1	378.33	393.78	412.97	398.72	375.29	369.56
2015-Q2	357.52	383.40	395.02	383.93	369.82	366.02
2015-Q3	364.54	380.13	395.06	383.49	367.52	363.34
2015-Q4	353.64	357.43	380.86	366.39	350.54	349.09
2016-Q1	301.58	296.62	340.09	337.36	314.22	343.48
$S_{r,t} = 0.5C_{r,t}$						
2015-Q1	378.48	393.74	413.07	398.41	375.62	369.76
2015-Q2	358.11	383.66	395.43	384.07	370.39	366.49
2015-Q3	365.09	380.55	395.53	383.81	368.16	363.92
2015-Q4	353.58	357.38	380.86	366.02	350.76	349.09
2016-Q1	301.80	296.66	340.26	337.16	314.53	343.41

Table 20: The model's out-of-sample predicted prices ($/ton) for the period 2015Q1–2016Q1 for different values of $S_{r,t}$.